# A Directed Spanning Tree Adaptive Control Framework for Time-Varying Formations

Dongdong Yue, Qi Li
School of Automation
Southeast University
Nanjing, China

Simone Baldi, Jinde Cao
School of Mathematics
Southeast University
Nanjing, China

Bart De Schutter
Delft Center for Systems and Control
Delft University of Technology
Delft, The Netherlands

Email: {d.yue, s.baldi, b.deschutter}@tudelft.nl

## 1 Introduction

In recent years, formation control of multi-agent systems has captured increasing attention due to applications in spacecraft formation flying, target seeking, search and rescue operations, to name a few [1]. By designing appropriate feasibility conditions, recent results on time-varying formation (TVF), and time-varying formation tracking (TVFT) have appeared as an extension to time-invariant formations. These designs rely on consensus-based methodologies to accomplish the formation in a *distributed* way (i.e. using local information only). If the design does not require any structural knowledge of the communication graph (e.g. the eigenvalues of the Laplacian matrix), then it is typically called *fully distributed*. Such designs have been proposed for undirected or detail-balanced/strongly-connected digraphs to address consensus, containment, or TVF problems. For more general digraphs, the directed spanning tree (DST) based distributed adaptive method was recently proposed to address consensus problems in a fully distributed way. However, fully distributed methods for TVF and TVFT problems are not yet available. In particular, in the presence of multiple leaders, it is impossible to have DSTs in the network since the leaders individually can not be influenced by any other agents. In this case, the problem of how to decouple the TVFT controller design and the network Laplacian spectra has not been addressed in the literature either. These observations motivate the work presented here.

## 2 Problem statements and control methodology

We consider TVF and TVFT problems for linear multi-agent systems over digraphs in a fully-distributed setting, i.e. without knowledge of the smallest eigenvalue of the Laplacian matrix associated to the communication graph. The solution to these problems relies on a unifying framework that generalizes the DST based adaptive method, which was originally limited to synchronization/consensus problems [2, 3]. For TVF without leaders, the control method consists in applying to each agent a distributed feedback control law with information of local state, local formation deviation, and neighboring formation deviations. The DST-based adaptive control approach leads to a novel class of feasibility conditions, which are more efficient to check than the feasibility conditions in the state of the art. For TVFT with leader(s), the agents are not only required to span some formation, but also supposed to track a single leader, or some convex combinations of multiple leaders. The control methodology then degenerates to a distributed feedback control law with information of local formation deviation and neighboring formation deviations. More specifically, a generalized DST-based adaptive control approach is proposed, providing a natural unifying framework for the DST-based adaptive methods in the presence of one or more leaders. By updating only the coupling weights of the edges in the tree, or the generalized tree, all of the control designs can be decoupled from the communication Laplacian spectra. Necessary and sufficient conditions for the existence of solutions to the formation problems are derived. Asymptotic convergence properties of the formation errors are proved via graph theory and Lyapunov analysis.

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