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Robust Neuro-adaptive Asymptotic Consensus for a Class of Uncertain Multi-agent Systems: An Edge-based Paradigm

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***Abstract*—In this brief, we investigate the adaptive consensus problem for a class of matching uncertain multi-agent systems in the presence of unmodelled nonlinearities and unknown disturbances. A robust dynamic controller comprising a linear feedback term, a discontinuous feedback term and a neural network approximation term is designed, where edge-based adaptive coupling strengths are introduced for both feedback terms while adaptive weights are designed for the neural network term. Unlike most existing neural-network-based control algorithms, where the consensus errors are ultimately uniformly bounded, the asymptotic consensus of the uncertain MAS is theoretically proved based on graph theory, nonsmooth analysis and Lyapunov theoretic approach. A noticeable character of the proposed method is that no global information of the underlying network is needed, such as the eigenvalues of the Laplacian matrix. In the end, a numerical example is presented to testify the theoretical results.**

***Index Terms*—Adaptive control, neural network, multi-agent systems, robust consensus.**

I. Introduction

Recent years, multi-agent systems (MASs) have shown the extraordinary talent as well as enormous potential in distributed computation and concurrent engineering, thus attracted many interests from various communities including control engineering, smart transportation, smart factory, etc [1]–[3]. Typically in cooperative control theory, the agents make a local decision, communicate their decisions with others, and repeat the process based on the new information received, finally leading the group to exhibit some special global behavior such as consensus, tracking, formation, to name a few. Consensus, as a foundational group behavior, is of great significance in many fields like distributed optimization [4] and power engineering [5].

Dealing with uncertainties has always been one of the major topics in control theory and applications. Various system uncertainties have been taken into account in the coordination analysis and synthesis of MASs in the past few years including time delays [6], stochastic disturbances [7], [8], parameter uncertainties [8], [9], switching topologies [10], unmodelled dynamics [11]–[13], and so on. In [6], fractional order observers have been designed for leader-following MASs with fractional-order followers and a second-order leader, with and without control-input delay, respectively. Distributed optimization for a class of nonlinear MASs subject to disturbances has been considered in [7], in which some sufficient conditions for semi-global and global distributed optimization while rejecting disturbances have been derived based on internal model method and Lyapunov theory. Distributed consensus tracking problem of nonlinear MASs with mismatched unknown parameters and uncertain disturbances has been addressed in [8] based on adaptive control. Distributed consensus output regulation problem of networked heterogeneous unknown linear systems on directed graphs has been concerned in [9], where adaptive control laws have been constructed to deal with the unknown subsystem parameters. In [10], several observer-based distributed protocols for consensus tracking of nonlinear MASs under directed switching topology have been proposed based on multiple Lyapunov function method. In [11], distributed cooperative tracking control of unknown higher-order affine nonlinear systems has been addressed, where neural networks (NNs) have been used to approximate the internal dynamics of the agents. However, the controller design relies on the underlying Laplacian matrix, thus can not be directly implemented for each agent. As an alternative to NN-based method, fully logic system has been introduced to approximate the unknown nonlinear dynamics existing in a second-order MASs in [12], where an adaptive fuzzy distributed controller has been proposed and analyzed to realize the consensus tracking. Recently in [13], distributed adaptive consensus for nonidentical matching nonlinear MAS has been concerned through an edge-based framework, where the infinite norms of the errors between the lumped matching uncertainties of neighboring agents are assumed to be uniformly bounded. However, this assumption is generally hard to be verify, thus limits the realistic application of the proposed method.

Motivated by the above discussions, this brief brings to-

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gether distributed cooperative control, NN approximation and nonsmooth analysis to solve the consensus problem for a class of uncertain MASs in the presence of unmodelled dynamics and external disturbances. In the proposed controller, neural networks with adaptive output weights are designed to approximate the unmodelled nonlinear functions, while nonsmooth techniques are introduced to compensate the unknown disturbances and nonlinear approximation errors. Meanwhile, edge-based time-varying coupling weights are developed via a Lyapunov theoretic approach to eliminate the dependence to the information of the underling network.

This brief is organized as follows. Section II presents some preliminaries and problem statement. Section III includes the main results of the topic. Section IV presents a numerical example to show the effectiveness of the proposed method and Section V finally concludes this paper.

Most of the notations in this paper are standard. Note that $\|x\|_1 = \sum_{i=1}^{n} |x_i|$ and $\|x\|_\infty = \max_{i\in\{1,\cdots,n\}} |x_i|$ are, respectively, the 1-norm and ∞-norm of a vector $x = (x_1, \cdots, x_n)^T \in \mathbb{R}^n$. Symbol $\otimes$ represents the Kronecker product. $Col(x_1, x_2, \cdots, x_n) = ({x_1}^T, {x_2}^T, \cdots, {x_n}^T)^T$ is the column vectorization of vectors (matrices) $x_1, x_2, \cdots, x_n$. $diag(\cdot)$ is the diagonalization operator and $sgn(\cdot)$ is the signum function defined component-wise.

II. Preliminaries and Problem Statement

A. Graph Theory

An undirected graph $\mathcal{G}$ is specified by a node set $\mathcal{V} = \{1, 2, \cdots, N\}$ and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, in which an undirected edge between node pair i and j is denoted by (i, j). An undirected path on $\mathcal{G}$ from vertex v_1 to vertex v_s corresponds a sequence of ordered edges of the form $(v_p, v_{p+1}), p = 1, 2, \cdots, s-1$. $\mathcal{G}$ is said to be connected if there exists a path between each pair of distinct nodes. The adjacency matrix $\mathcal{A} = (a_{ij}) \in \mathbb{R}^{N\times N}$ of $\mathcal{G}$ is the symmetric $\{0, 1\}$-matrix, such that $a_{ij} = 1$ if and only if $(i, j) \in \mathcal{E}$. The incidence matrix $D = (D_{ik}) \in \mathbb{R}^{N\times|\mathcal{E}|}$ of $\mathcal{G}$ (with a particular orientation) is the $\{0, \pm1\}$-matrix, such that $D_{ik} = 1$ if the vertex i is the head of the edge k, -1 if i is the tail of k, and 0 otherwise. The Laplacian matrix $\mathcal{L} = (l_{ij}) \in \mathbb{R}^{N\times N}$ of $\mathcal{G}$ is defined as: $l_{ij} = -a_{ij}, i \neq j$, and $l_{ii} = \sum_{j=1,j\neq i}^{N} a_{ij}, i = 1, 2, \cdots, N$. Then one has $\mathcal{L} = DD^T$. In this paper, $\mathcal{G}$ is always a simple graph, i.e., a graph without multiple edges and self-loops.

Assumption 1: The communication topology $\mathcal{G}$ among the N agents is undirected and connected.

Lemma 1: Under Assumption 1, the laplacian matrix $\mathcal{L}$ has a simple eigenvalue 0 with $\mathbf{1}$ as a corresponding right eigenvector, and all the other eigenvalues have positive real parts.

B. Nonsmooth Analysis

Consider the following differential equation with a discontinuous right-hand side:

$$\dot{z} = g(z, t) \tag{1}$$

where $g : \mathbb{R}^n \times [0, +\infty) \to \mathbb{R}^n$ is a Lebesgue measurable function and is locally essentially bounded. $z(t)$ is called a Filippov solution of (1) over $[t_a, t_b]$, if $z(t)$ is absolutely continuous on $[t_a, t_b]$ and satisfies that $\dot{z} \in \mathcal{K}[g](z, t)$, for almost all t. Here $\mathcal{K}[g](z, t) \triangleq \bigcap_{\alpha>0} \bigcap_{\mu(\bar{N})=0} \bar{co}\{g(B(z, \alpha) - \bar{N}, t)\}$, in which $\bigcap_{\mu(\bar{N})=0}$ represents the intersection over all sets $\bar{N}$ of Lebesgue measure zero, $\bar{co}\{\cdot\}$ is the convex closure operation and $B(z, \alpha)$ denotes the open ball of radius α centered at z.

Suppose function $V(z) : \mathbb{R}^n \to \mathbb{R}$ is locally Lipschitz continuous on $z(t)$, then the set-valued Lie derivative of $V(z)$ along (1) is defined as

$$\dot{\tilde{V}}(z(t)) = \bigcap_{\xi\in\partial V(z(t))} \xi^T \mathcal{K}[g](z(t), t),$$

where $\partial V(z)$ represents the Clarke's generalized gradient of V at z which is defined as $\partial V(z) = \bar{co}\{\lim_{i\to\infty} \nabla V(z_i)|z_i \to z, z_i \notin \Omega_u \bigcup \bar{N}\}$. Here, Ω_u denotes the set of Lebesgue measure zero where ∇V does not exist and $\bar{N}$ represents an arbitrary set of Lebesgue measure zero.

C. Problem Statement

Consider a MAS of N agents described by

$$\dot{x}_i(t) = Ax_i(t) + B(u_i(t) + f_i(x_i(t)) + \omega_i(t)) \qquad i = 1, 2, \cdots, N, \tag{2}$$

where $x_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}^m$ are, respectively, the state and control input of agent i. A, B are known constant matrices with compatible dimensions such that (A, B) is stabilizable. $f_i(x_i) \in \mathbb{R}^m$ is unknown matching input nonlinearity which is assumed to be smooth, and $\omega_i(t) \in \mathbb{R}^m$ denotes the unknown matching disturbance of agent i which satisfies the following assumption:

Assumption 2: For each agent i, there exists constant $\omega_{iM} > 0$, such that $\|\omega_i(t)\|_\infty \leq \omega_{iM}$, $i = 1, \cdots, N$.

In this paper, NNs are utilized to directly compensate the effect of unknown nonlinearities in the system. Since the unknown function $f_i(x_i)$ is smooth, it can thus be approximated by NNs according to the Stone-Weierstrass theorem [14]. Concretely,

$$f_i(x_i) = W_i^T S_i(x_i) + \epsilon_i(x_i) \quad \forall x_i \in \Omega_i, \tag{3}$$

where $W_i \in \mathbb{R}^{s\times m}$ is an ideal weight matrix which is unknown but bounded by $\|W_i\|_F \leq W_{iM}$, $S_i(x_i) : \mathbb{R}^n \mapsto \mathbb{R}^s$ is a vector collection of basis functions of the form $S_i(x_i) = (S_{i1}(x_i), \cdots, S_{is}(x_i))^T$ and satisfies $\|S_i\| \leq S_{iM}$. $\epsilon_i(t)$ is the approximation error vector satisfying $\|\epsilon_i\| \leq \epsilon_{iM}$ and Ω_i is a sufficiently large compact set. These boundbess conditions are fairly standard when processing approximation tasks with NNs [11], [15]–[17].

Remark 1: Noted that the agents are actually heterogeneous as the matching uncertainties $f_i(x_i)$ and $\omega_i(t)$ are allowed to be different. Besides, the constants ω_{iM}, W_{iM}, S_{iM} and ϵ_{iM} could be unknown for the controller design, which serve only for theoretical analysis.

The control object is to design proper input $u_i(t)$ for each agent i to realize robust asymptotic consensus, i.e.,

$\lim_{t\to\infty} \|x_i - x_j\| = 0, \forall i, j \in \mathcal{V}$, despite of the unmodelled dynamics and disturbances in the MAS.

III. Robust Neuro-adaptive Asymptotic Consensus via Edge-based Distributed Protocol

A. Controller Description

To keep the notations simple, define the consensus error of agent i as $e_i = x_i - \frac{1}{N}\sum_{j=1}^{N} x_j$. The proposed adaptive strategy for node i is designed as

$$\begin{aligned} u_i =& \sum_{j\in\mathcal{N}_i} c_{ij}(t)F(x_i - x_j) \\ &+ \sum_{j\in\mathcal{N}_i} d_{ij}(t)sgn(F(x_i - x_j)) \\ &- \hat{W}_i^T(t)S_i(x_i) \\ \dot{c}_{ij}(t) =& \kappa_{ij}(x_i - x_j)^T\Gamma(x_i - x_j) \quad j \in \mathcal{N}_i \\ \dot{d}_{ij}(t) =& \nu_{ij}\|F(x_i - x_j)\|_1 \qquad j \in \mathcal{N}_i \\ \dot{\hat{W}}_i =& \tau_i[S_i(x_i)e_i^T P^{-1}B - \sigma_i(\hat{W}_i - \overline{W}_i(t))] \\ \dot{\overline{W}}_i =& \sigma_i\pi_i(\hat{W}_i - \overline{W}_i), \end{aligned} \tag{4}$$

where F and Γ are feedback gain matrices to be designed, $c_{ij}(t)$, $d_{ij}(t)$, $(i, j) \in \mathcal{E}$ are the dynamic coupling strengths. κ_{ij} and ν_{ij} are small constant gains and $\hat{W}_i(t)$ is the estimation of the unknown weight matrix W_i and $\overline{W}_i(t)$ is the pseudo ideal weight matrix introduced for agent i, τ_i, σ_i and π_i are positive scalars.

B. Main Results and Theoretical Analysis

In this next, we study the consensus problem of the MAS (2) under protocol (4).

Let $x = col(x_1, x_2, \cdots, x_N)$, $e = col(e_1, e_2, \cdots, e_N)$, and $\Xi = \boldsymbol{I}_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T$. Then we have $e = (\Xi \otimes \boldsymbol{I}_n)x$. Note that Ξ has a simple eigenvalue 0 and $N-1$ multiple eigenvalue 1 and there hold under Assumption 1 that $\mathcal{L}\Xi = \Xi\mathcal{L} = \mathcal{L}$, $\Xi^2 = \Xi$ and $\Xi D = D$.

Initialize $c_{ij}(0) = 0$ and $d_{ij}(0) = 0$, it follows by selecting $\kappa_{ij} = \kappa_{ji}$ and $\nu_{ij} = \nu_{ji}$ in (4) and Assumption 1 that $c_{ij}(t) = c_{ji}(t)$ and $d_{ij}(t) = d_{ji}(t)$, $\forall t \geq 0$ and $c_{ij}(t) = d_{ij}(t) = 0$ if $a_{ij} = 0$. Thus, this protocol actually attaches two independent coupling strength $c_{ij}(t)$ and $d_{ij}(t)$ to each edge $(i, j) \in \mathcal{E}$. Denote $q = |\mathcal{E}|$ and $\mathcal{E}_1, \cdots, \mathcal{E}_q$ the edges in $\mathcal{E}$.

Under (3) and (4), the closed loop dynamics of (2) can be written as

$$\begin{aligned} \dot{x}_i =& Ax_i + B(\sum_{j\in\mathcal{N}_i} c_{ij}(t)F(x_i - x_j) \\ &+ \sum_{j\in\mathcal{N}_i} d_{ij}(t)sgn(F(x_i - x_j))) \\ &- B(\tilde{W}_i^T S_i(x_i) - \epsilon_i - \omega_i). \end{aligned} \tag{5}$$

We claim that

$$\sum_{j\in\mathcal{N}_i} c_{ij}(t)(x_i - x_j) = \sum_{j=1}^{N}(D\mathcal{M}_1 D)_{ij}x_j \tag{6}$$

where $D \in \mathcal{R}^{N\times q}$ is the incidence matrix of $\mathcal{G}$ and $\mathcal{M}_1 = diag(c_{ij}(t))$ in which $c_{ij}(t)$ are fed in the order of edge index. In fact,

$$\begin{aligned} &\sum_{j=1}^{N}(D\mathcal{M}_1 D)_{ij}x_j \\ =&\sum_{j=1}^{N}(\sum_{k=1}^{q} D_{ik}c_{\mathcal{E}_k}D_{jk})x_j \\ =&\sum_{k=1}^{q} D_{ik}c_{\mathcal{E}_k}D_{ik}x_i + \sum_{j=1,j\neq i}^{N}\sum_{k=1}^{q} D_{ik}c_{\mathcal{E}_k}D_{jk}x_j \\ =&\sum_{j\in\mathcal{N}_i} c_{ij}(t)x_i - \sum_{j\in\mathcal{N}_i} c_{ij}(t)x_j \\ =&\sum_{j\in\mathcal{N}_i} c_{ij}(t)(x_i - x_j) \end{aligned} \tag{7}$$

where we have also used $c_{\mathcal{E}_k}$ to represent the k-th edge coupling strength when the endpoint indexes are unconcerned. Similarly, by letting $\mathcal{M}_2 = diag(d_{ij}(t))$, it follows that

$$\begin{aligned} &\sum_{j\in\mathcal{N}_i} d_{ij}(t)sgn(F(x_i - x_j)) \\ =&\sum_{k=1}^{q}(D\mathcal{M}_2)_{ik}sgn(\sum_{j=1}^{N} D_{jk}Fx_j). \end{aligned} \tag{8}$$

Then, under the facts (6) and (8), (5) can be written in compact form as

$$\begin{aligned} \dot{x} =& (\boldsymbol{I}_N \otimes A)x + (D\mathcal{M}_1 D^T \otimes BF)x \\ &+ (D\mathcal{M}_2 \otimes B)sgn((D^T \otimes F)x) \\ &- (\boldsymbol{I}_N \otimes B)(\tilde{W}^T S(x) - \epsilon - \omega). \end{aligned} \tag{9}$$

where $\tilde{W} = diag(\tilde{W}_1, \cdots, \tilde{W}_N)$, $S(x) = col(S_1(x_1), \cdots, S_N(x_N))$, $\epsilon = col(\epsilon_1, \cdots, \epsilon_N)$, $\omega = (\omega_1, \cdots, \omega_N)$ and $\tilde{W}_i = \hat{W}_i - W_i$, $i = 1, \cdots, N$. Then there exist positive constants W_M, S_M and ϵ_M, such that

$$\|W\|_F \leq W_M, \quad \|S(x)\|_F \leq S_M, \quad \|\epsilon\|_\infty \leq \epsilon_M. \tag{10}$$

Moreover, there exist ω_M such that $\|\omega\|_\infty \leq \omega_M$ by Assumption 2.

Thus, we can obtain the dynamics of error system e as

$$\begin{aligned} \dot{e} =& (\boldsymbol{I}_N \otimes A + D\mathcal{M}_1 D^T \otimes BF)e \\ &+ (D\mathcal{M}_2 \otimes B)sgn((D^T \otimes F)e) \\ &- (\Xi \otimes B)(\tilde{W}^T S(x) - \epsilon - d). \end{aligned} \tag{11}$$

The following theorem summarizes the main contribution of this paper.

Theorem 1: Under the Assumptions 1 and 2, the asymptotic consensus problem of MAS (2) can be solved under dynamic neuro-adaptive protocol (4) with $F = -B^T P^{-1}$ and $\Gamma = P^{-1}BB^T P^{-1}$, where $P > 0$ is a solution to the following linear matrix inequality (LMI):

$$AP + PA^T - \eta BB^T + \theta P \leq 0 \tag{12}$$

for some scalars $\eta,\ \theta > 0$. Moreover, each estimated NN weight matrix $\hat{W}_i$ converges to the corresponging pesudo ideal weight matrix $\overline{W}_i$ and each coupling gain $c_{ij}(t)$ as well as $d_{ij}(t)$ converges to some finite value.

Proof. Note that the dynamics equation of the error system (11) has discontinuous right-hand side, so the stability of which will be analysed according to differential inclusions and nonsmooth theory [18]. Thanks to the measurability and essentially boundness of the signum function, the Filippov solution of (11) exists. In terms of differential inclusions, (11) can be written as

$$\begin{aligned}\dot{e} \in^{a.e.} &\mathcal{K}[(\boldsymbol{I}_N \otimes A + D\mathcal{M}_1 D^T \otimes BF)e \\ &+ (D\mathcal{M}_2 \otimes B)sgn((D^T \otimes F)e) \\ &- (\Xi \otimes B)(\tilde{W}^T S(x) - \epsilon - d)], \end{aligned} \tag{13}$$

where $a.e.$ represents 'almost everywhere'.

Consider the following Lyapunov function candidate:

$$\begin{aligned}V =& e^T(\Xi \otimes P^{-1})e \\ &+ \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} \frac{1}{2\kappa_{ij}}(c_{ij}(t)-\tilde{c})^2 + \frac{1}{2\nu_{ij}}(d_{ij}(t)-\tilde{d})^2 \\ &+ \sum_{i=1}^{N} tr(\frac{1}{\tau_i}\tilde{W}_i^T\tilde{W}_i + \frac{1}{\pi_i}\tilde{\bar{W}}_i^T\tilde{\bar{W}}_i) \end{aligned} \tag{14}$$

where $\tilde{c}$ and $\tilde{d}$ are two positive constants to be determined later, $\tilde{\bar{W}}_i = \bar{W}_i - W_i$ and $tr(\cdot)$ is the trace operator.

By using the property of $\mathcal{K}$, the set-valued Lie derivative of V along (11) can be obtained as

$$\begin{aligned}\dot{\tilde{V}} =& 2e^T(\Xi \otimes P^{-1}A + D\mathcal{M}_1 D^T \otimes P^{-1}BF)e \\ &- 2e^T(\Xi \otimes P^{-1}B)(\tilde{W}^T S(x) - \epsilon - d) \\ &+ \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} \frac{1}{\kappa_{ij}}(c_{ij}(t)-\tilde{c})\dot{c}_{ij} + \frac{1}{\nu_{ij}}(d_{ij}(t)-\tilde{d})\dot{d}_{ij} \\ &+ 2\sum_{i=1}^{N} tr(\frac{1}{\tau_i}\tilde{W}_i^T\dot{\tilde{W}}_i + \frac{1}{\pi_i}\tilde{\bar{W}}_i^T\dot{\tilde{\bar{W}}}_i) \\ &+ \mathcal{K}[2e^T(D\mathcal{M}_2 \otimes P^{-1}B)sgn((D^T \otimes F)e)]. \end{aligned} \tag{15}$$

Substituting $F = -B^T P^{-1}$ and $\dot{W}_i$, $\dot{\overline{W}}_i$ in (4) into above gives

$$\begin{aligned}\dot{\tilde{V}} =& 2e^T(\Xi \otimes P^{-1}A - D\mathcal{M}_1 D^T \otimes P^{-1}BB^TP^{-1})e \\ &- 2e^T(\Xi \otimes P^{-1}B)(\tilde{W}^T S(x) - \epsilon - d) \\ &+ \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} \frac{1}{\kappa_{ij}}(c_{ij}(t)-\tilde{c})\dot{c}_{ij} + \frac{1}{\nu_{ij}}(d_{ij}(t)-\tilde{d})\dot{d}_{ij} \\ &+ 2\sum_{i=1}^{N} tr(\tilde{W}_i^T S_i(x_i)\delta_i^T P^{-1}B) \\ &- 2\sum_{i=1}^{N} tr(\sigma_i(\hat{W}_i - \overline{W}_i)^T(\hat{W}_i - \overline{W}_i)) \\ &- \mathcal{K}[2e^T(D\mathcal{M}_2 \otimes P^{-1}B)sgn((D^T \otimes B^TP^{-1})e)]. \end{aligned} \tag{16}$$

Note that

$$\begin{aligned}&2e^T(D\mathcal{M}_2 \otimes P^{-1}B)sgn((D^T \otimes B^TP^{-1})e) \\ &= \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} d_{ij}(t)\|B^TP^{-1}(e_i - e_j)\|_1, \end{aligned} \tag{17}$$

then the set-valued Lie derivative of $V(t)$ is a singleton. Furthermore, one has

$$e^T(\Xi \otimes P^{-1}B)(\tilde{W}^T S(x)) = \sum_{i=1}^{N} tr(\tilde{W}_i^T S_i(x_i) e_i^T P^{-1}B), \tag{18}$$

and

$$\begin{aligned}&2e^T(D\mathcal{M}_1 D^T \otimes P^{-1}BB^TP^{-1})e \\ &= \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} c_{ij}(t)(e_i - e_j)^T P^{-1}BB^TP^{-1}(e_i - e_j). \end{aligned} \tag{19}$$

It thus follows from (17), (18) and (19) that

$$\begin{aligned}\dot{\tilde{V}} =& 2e^T(\Xi \otimes P^{-1}A)e \\ &- \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} c_{ij}(t)(e_i - e_j)^T P^{-1}BB^TP^{-1}(e_i - e_j) \\ &+ 2e^T(\Xi \otimes P^{-1}B)(\epsilon + d) \\ &+ \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} \frac{1}{\kappa_{ij}}(c_{ij}(t)-\tilde{c})\dot{c}_{ij} + \frac{1}{\nu_{ij}}(d_{ij}(t)-\tilde{d})\dot{d}_{ij} \\ &- 2\sum_{i=1}^{N} tr(\sigma_i(\hat{W}_i - \overline{W}_i)^T(\hat{W}_i - \overline{W}_i)) \\ &- \sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} d_{ij}(t)\|B^TP^{-1}(e_i - e_j)\|_1. \end{aligned} \tag{20}$$

It follows by substitute $\dot{c}_{ij}$ and $\dot{d}_{ij}$ in (4) into (20) and meanwhile noting $\Gamma = P^{-1}BB^TP^{-1}$ that

$$\begin{aligned}\dot{\tilde{V}} \le& 2e^T(\Xi \otimes P^{-1}A)e \\ &+ 2e^T(\Xi \otimes P^{-1}B)(\epsilon + d) \\ &- \tilde{c}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} (e_i - e_j)^T P^{-1}BB^TP^{-1}(e_i - e_j) \\ &- \tilde{d}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} \|B^TP^{-1}(e_i - e_j)\|_1. \end{aligned} \tag{21}$$

Since

$$
\begin{aligned}
&2e^T(\Xi\otimes P^{-1}B)(\epsilon+d)\\
\le&2\|(\Xi\otimes B^TP^{-1})e\|_1(\|\epsilon\|_\infty+\|d\|_\infty)\\
\le&2(\epsilon_M+\omega_M)\|(\Xi\otimes B^TP^{-1})e\|_1\\
\le&\frac{2(\epsilon_M+\omega_M)}{N}\sum_{i=1}^N\sum_{j=1}^N\|B^TP^{-1}(e_i-e_j)\|_1\\
\le&2(\epsilon_M+\omega_M)\max_{i=1,\cdots,N}\Big\{\sum_{j=1}^N\|B^TP^{-1}(e_i-e_j)\|_1\Big\}\\
\le&(\epsilon_M+\omega_M)(N-1)\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\|B^TP^{-1}(e_i-e_j)\|_1, \quad (22)
\end{aligned}
$$

then by choosing $\tilde{d}>(\epsilon_M+\omega_M)(N-1)$, it follows from (21) and (22) that

$$
\begin{aligned}
\dot{\tilde{V}}\le&2e^T(\Xi\otimes P^{-1}A)e\\
&-\tilde{c}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}(e_i-e_j)^TP^{-1}BB^TP^{-1}(e_i-e_j)\\
&-(\tilde{d}-(\epsilon_M+\omega_M)(N-1))\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\|B^TP^{-1}(e_i-e_j)\|_1\\
\le&2e^T(\Xi\otimes P^{-1}A)e\\
&-\tilde{c}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}(e_i-e_j)^TP^{-1}BB^TP^{-1}(e_i-e_j)\\
=&\bar{e}^T(\Xi\otimes(AP+PA^T)-2\tilde{c}\mathcal{L}\otimes BB^T)\bar{e}\\
\le&\bar{e}^T(\Xi\otimes(AP+PA^T-2\tilde{c}\lambda_2BB^T))\bar{e} \quad (23)
\end{aligned}
$$

where $\bar{e}=(\boldsymbol{I}_N\otimes P^{-1})e$. Select $\tilde{c}>\frac{\eta}{2\lambda_2}$, then from (12), we have

$$
\dot{\tilde{V}}\le-\theta\bar{e}^T(\Xi\otimes P)\bar{e}=-\theta e^T(\Xi\otimes P^{-1})e \quad (24)
$$

which implies that $V(t)$ is nonincreasing. Then it follows that all signals in $V(t)$, e, c_{ij}, d_{ij}, $\tilde{W}_i$ and $\tilde{\overline{W}}_i$, are uniformly bounded. Since $V(t)\ge 0$ and is nonincreasing, it thus has a finite limit V^∞ as $t\to\infty$. Noted that $\dot{e}$ is also uniformly bounded in light of (10) and (11), then $\theta e^T(t)(\mathcal{L}\otimes P^{-1})e(t)$ is uniformly continuous. By utilizing Barbalat's Lemma, one has $\lim_{t\to\infty}\theta e^T(t)(\Xi\otimes P^{-1})e(t)=0$, which guarantees $\lim_{t\to\infty}e(t)=0$.

On the other hand, denote $\hat{W}_i^e=\hat{W}_i-\overline{W}_i$, then it follows (4) that

$$
\dot{\hat{W}}_i^e=-\sigma_i(\tau_i+\pi_i)\hat{W}_i^e+\tau_iS_i(x_i)e_i^TP^{-1}B. \quad (25)
$$

Since $\lim_{t\to\infty}e(t)=0$, $S_i(x_i)$ is uniformly bounded, σ_i, τ_i and π_i are given positive constants. By taking $\tau_iS_i(x_i)e_i^TP^{-1}B$ as the control input, system (25) is input to state stable. One may then conclude that $\lim_{t\to\infty}\hat{W}_i^e=0$, i.e., $\lim_{t\to\infty}(\hat{W}_i-\overline{W}_i)=0$. Moreover, $c_{ij}(t)$ and $d_{ij}(t)$ are monotonically increase as $\Gamma>0$, κ_{ij} and $\nu_{ij}>0$, which guarantees each of them will converge to some finite steady values in consider of their boundness. This completed this proof. ∎

Corollary 1: Under the Assumptions 1 and 2, the asymptotic consensus problem of MAS (2) can be solved under static neuro-adaptive protocol

$$
\begin{aligned}
u_i=&c\sum_{j\in\mathcal{N}_i}F(x_i-x_j)+d\sum_{j\in\mathcal{N}_i}sgn(F(x_i-x_j))\\
&-\hat{W}_i^T(t)S_i(x_i)\\
\dot{\hat{W}}_i=&\tau_i[S_i(x_i)e_i^TP^{-1}B-\sigma_i(\hat{W}_i-\overline{W}_i(t))]\\
\dot{\overline{W}}_i=&\sigma_i\pi_i(\hat{W}_i-\overline{W}_i) \quad (26)
\end{aligned}
$$

with $F=-B^TP^{-1}$, $c>\frac{\eta}{2\lambda_2}$ and $d>(\epsilon_M+\omega_M)(N-1)$, where P and η are defined in (12).

Remark 2: It should be noted that the perspective of this paper is different from that of [19] and [13], where some boundness assumptions are applied to the unknown matched nonlinearities. In this set-up, NNs are employed to directly compensate the nonlinear uncertainties without assuming their boundness.

Remark 3: In comparison with traditional NN-based control algorithms, where the consensus errors are usually proved to be ultimately uniformly bounded (UUB) [15], [16], [20]), the introduction of the ideal weight matrices in (4) enables us to drive the consensus errors to zero asymptotically. This superiority makes it possible to design edge-based adaptive strategies with the help of Barbalat's Lemma. Even though, it is still unclear whether the estimated NN weight $\hat{W}_i$ converges to its corresponding ideal weight matrix W_i [17].

Remark 4: Note that the Lyapunov function V in Theorem 1 does not depend on any information of the underlying network $\mathcal{G}$. Thus, it is easy to extend the results to the MAS under switching communication topologies as long as each possible topology is connected and undirected.

IV. Numerical examples

In this section, a numerical example is presented to verify the obtained theoretical results. Consider a MAS with 6 agents, whose dynamics are governed by (2) with $x_i=(x_{i1},x_{i2})^T$,

$$
A=\begin{pmatrix}0&1\\-1&2\end{pmatrix},\quad B=\begin{pmatrix}0\\1\end{pmatrix},
$$

$f_i(x_i)={x_{i1}}^2\cos(i+x_{i2})$ and $d_i(t)=0.1\sin(it)$. Considering Remark 4, the communication graph among the agents are assumed randomly switching every 0.1 second between the two possibly topologies depicted in Fig. 1.

To verify Theorem 1, choose $\eta=2$, $\theta=1$ and solve LMI (12) giving that $P=\begin{pmatrix}0.0816&-0.0839\\-0.0839&0.2855\end{pmatrix}$, then it follows that

$$
F=-(5.1621,5.0202),\quad \Gamma=\begin{pmatrix}26.6477&25.9150\\25.9150&25.2024\end{pmatrix}.
$$

Let $\kappa_{ij}=\nu_{ij}=0.01$, $\tau_i=\pi_i=10$ and $\sigma_i=0.5$ in protocol (4). The initial positions of the agents are chosen randomly following normal distribution with standard deviation of 2. For

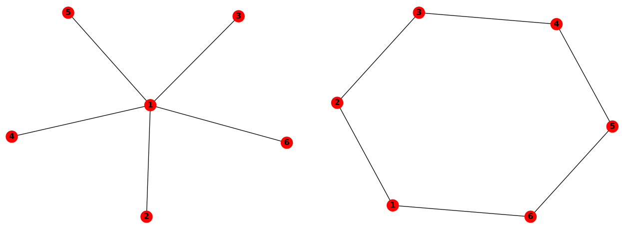

Fig. 1. Two possible communication topologies.

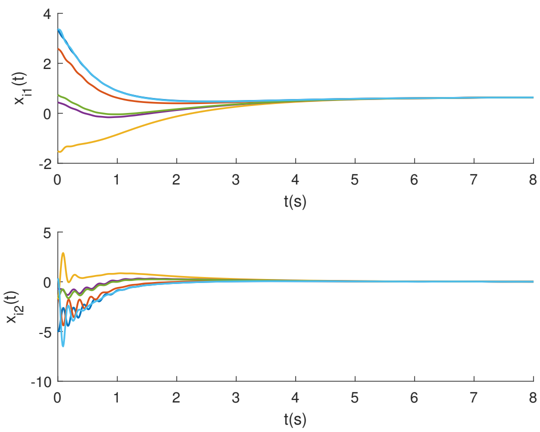

Fig. 2. States of agents $x_i(t)$, $i = 1, \cdots, 6$.

each agent, 60 hidden neurons with Sigmoid activation functions are assigned to approximate the nonlinearities. After the simulation, the state trajectories of all agents are visualized in Fig. 2. The component-wise values of pseudo converge errors $\hat{W}_i^e$ and the NN approximation errors $\hat{W}_i^T(t)S_i(x_i) - f_i(x_i)$ are provided in Fig. 3. The profiles of coupling weights c_{ij} and d_{ij} are given in Fig.4, where each adaptive variable for all 10 possible communication channels remains steady after about 5 seconds. Finally, the control efforts are shown in Fig. 5. It can be seen that the numerical example validates the theoretical results.

V. Conclusion

In this paper, robust adaptive consensus for a class of heterogeneous uncertain MASs subject to unmodelled dynamics and external disturbances is addressed. Without knowing any numerical information of the underlying communication network, an edge-based distributed framework is constructed while neural networks are assigned for the agents to approximate their unmodelled dynamics. Asymptotic consensus among agents is rigorously proved with the help of nonsmooth analysis and Barbalat's lemma before an illustrative example presented to validate the theoretical results. Note that the NN weight adaption laws depend on global states information of the system, which calls for further optimization of the control strategy. Future focus along the same line would also be the counterparts under directed communication topologies and observer-based mechanisms.

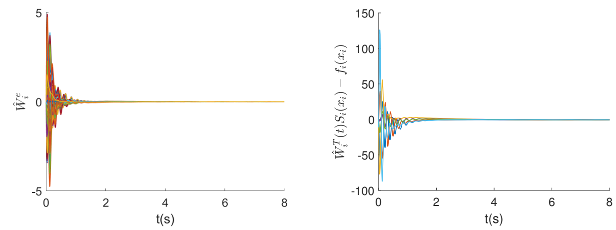

Fig. 3. Component-wise values of $\hat{W}_i^e$, and NN approximation errors $\hat{W}_i^T(t)S_i(x_i) - f_i(x_i)$, $i = 1, \cdots, 6$.

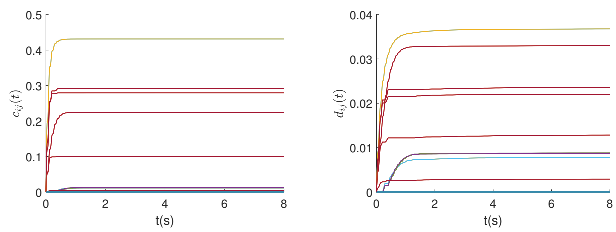

Fig. 4. Coupling weights $c_{ij}(t)$, and $d_{ij}(t)$, $i, j = 1, \cdots, 6$.

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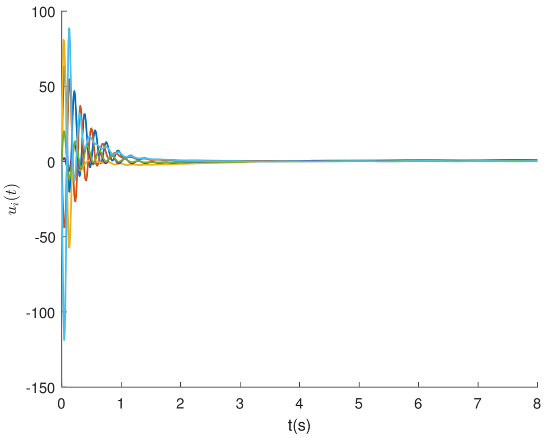

Fig. 5. Control inputs $u_i(t)$, $i = 1, \cdots, 6$.

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