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Distributed Adaptive Consensus via Event-triggered Sampling: An Edge-based Method

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***Abstract*—This paper addresses distributed adaptive consensus control of general linear multiagent systems (MASs) under event-triggered sampling mechanism. We propose a novel edge-based method in which, for each communication link, an adaptive coupling gain channel and a sampling triggering function are co-designed. The benefits are that the proposed method requires neither the global knowledge of the network eigenvalues for gain selection, nor continuously state sampling for control.**

***Index Terms*—distributed control, consensus, adaptive control, event-triggered control**

I. Introduction

Consensus of multiagent systems (MASs) serves as building blocks for several complex engineering as well as scientific problems, e.g., formation of UAVs [1], distributed optimization [2], graph signal processing [3], etc. Due to these promising applications, consensus control has always been an active topic in the control community.

Beyond distributed control of MASs, the concept of *fully-distributed control* (also referred to as *distributed adaptive control*) has been well established in recent years [4]–[7], where the term *adaptive* refers to the introduction of some adaptive coupling gains between the agents. An excellent feature of such a strategy is that the algebraic connectivity information (represented by the global Laplacian eigenvalues) is not needed, whereas standard distributed strategies require this prior knowledge for gain selection of the controllers.

At the same time, event-triggered control has been a popular method to save the cost of continuously communicating and updating the controllers in MASs [8]–[11]. In event-triggered control systems, the controller updates only when some carefully designed *events* are *triggered*, which is usually more economic in terms of communication cost. We refer the readers to some recent surveys [12], [13] for the developments and applications of event-triggered control in MASs.

A natural question emerges that whether adaptive coupling gains and event-triggered sampling mechanisms can be designed simultaneously, so as to achieve fully-distributed event-triggered control. Fortunately, we have seen several recent works that provide positive answers. For instance, [14], [15] have addressed consensus and formation problems, respectively, by fully-distributed event-triggered control where some edge-based coupling gains are designed. Note that in [15], the authors require two triggering functions for a single communication link, which may be unpractical. Most recently, [16] has proposed a class of node-based fully-distributed event-triggered consensus control for MASs over a static network, where the adaptive coupling gains and triggering functions are both designed in terms of the nodes.

Motivated by the above discussions, this brief aims to provide a novel edge-based fully-distributed event-triggered consensus control method for general linear MASs. In other words, an adaptive coupling gain channel and a sampling triggering function are co-designed for each communication link. We show that the proposed method guarantees closed-loop stability and Zeno-free property. Also, this edge-based method is applicable to the case of MASs over switching graphs, which is in line with conventional edge-based fully-distributed control framework [4].

Our notations are fairly standard and definitions are given if necessary. Let us now close this section by recalling some notations in algebraic graph theory: Denote by $\mathcal{G}(\mathcal{V},\mathcal{E})$ is an undirected (simple) graph with a node set $\mathcal{V}=\{1,\cdots,N\}$ and an edge set $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$. An element $(i,j)\in\mathcal{E}$ indicates that there is a undirected edge between node i and j, in which case j is a neighbor of i, denoted by $j\in\mathcal{N}_i$, and vice versa. The incidence matrix of $\mathcal{G}$ (with arbitrarily settled edge orientations) is $E=(E_{ik})\in\mathbb{R}^{|\mathcal{V}|\times|\mathcal{E}|}$, where $E_{ik}=1$ if the vertex i is the head of the edge indexed by k, -1 if i is the tail of k, and 0 otherwise. The Laplacian matrix of $\mathcal{G}$ is $\mathcal{L}=EE^T$. A path of $\mathcal{G}$ is an ordered series of edges connecting two nodes. The graph $\mathcal{G}$ is connected, if there exists a path between each pair of distinct nodes. The Laplacian matrix of a connected graph is semi-definite with a single zero eigenvalue, in which case we use $\lambda_2(\mathcal{L})$ to denote its smallest nonzero eigenvalue.

II. Preliminaries

Consider a MAS with N ($N\geq 2$) linear time-invariant agents as

$$\dot{x}_i(t)=Ax_i(t)+Bu_i(t),\qquad i=1,\cdots,N, \tag{1}$$

where $x_i\in\mathbb{R}^n$ and $u_i\in\mathbb{R}^m$ are, respectively, the state variable and control input of agent i. The matrix pair (A,B) are known, of compatible dimensions, and stabilizable. The agents interact with each other over an undirected graph $\mathcal{G}(\mathcal{V},\mathcal{E})$. The MAS (1) is said to achieve consensus if, for any given bounded

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initial states $x_i(0)$, there holds $\lim_{t\to\infty}\|x_i(t)-x_j(t)\|=0$, $\forall i,j\in\mathcal{V}$.

Our objective is to attain consensus with *distributed adaptive event-triggered* strategies. In other words, the desired controller for each agent should solely depend on relative information of its neighbors sampled according to some triggering rule.

III. Main Results

In this section, we propose a novel edge-based distributed adaptive event-triggered controller, which can be seen a counterpart of the node-based algorithm in [16]. Hereinafter, the explicit dependence of the signals on the time stamp t will omitted if clear from the context.

For simplicity, let us denote $x_{ij}(t)\triangleq x_i(t)-x_j(t)$. Based on sampled relative errors w.r.t the neighbors $j\in\mathcal{N}_i$, the control law for agent $i\in\mathcal{V}$ is proposed as

$$\begin{aligned}u_i&=K\sum_{j\in\mathcal{N}_i}c_{ij}\tilde{x}_{ij}\\ \dot{c}_{ij}&=\rho_{ij}\tilde{x}_{ij}^T\Gamma\tilde{x}_{ij},\end{aligned}\tag{2}$$

where K is a feedback gain matrix and $\rho_{ij}\in\mathbb{R}^+$; c_{ij} is an adaptive coupling gain; $\tilde{x}_{ij}(t)\triangleq x_{ij}(t_k^{ij})$ for $t\in[t_k^{ij},t_{k+1}^{ij})$ is the sampled relative error between agent i and j, i,e., both ends of the edge (i,j). The sampling instants t_k^{ij} $(k\in\mathbb{N})$ are determined only by agent i and j, via cooperatively monitoring of a triggering function. Specifically, for each edge $(i,j)\in\mathcal{E}$, let $t_0^{ij}=0$ and $t_{k+1}^{ij}=\inf_{t>t_k^{ij}}\{f_{ij}(\epsilon_{ij},x_{ij},c_{ij},t)=0\}$, where the triggering function is given by

$$f_{ij}=\epsilon_{ij}^T\Gamma\epsilon_{ij}-\omega x_{ij}^T\Gamma x_{ij}-\frac{\theta}{c_{ij}}\mathrm{e}^{-\eta t}\tag{3}$$

where $\epsilon_{ij}\triangleq\tilde{x}_{ij}-x_{ij}$ and $\omega,\theta,\eta\in\mathbb{R}^+$. From the symmetry structure of f_{ij}, one knows that $t_k^{ij}\equiv t_k^{ji}$ for any $k\in\mathbb{N}$ and any edge $(i,j)\in\mathcal{E}$.

Remark 1: Before moving forward, it should be mentioned that the timely neighboring information of x_{ij} is needed for the triggering function f_{ij}, which means that continuously monitoring of this relative error is required. This problem is common to most literature and can be solved using self-triggering control in a similar way as [16], [17].

The design process of the control parameters is summarized in Algorithm 1.

Algorithm 1

1) Choose κ_1, $\kappa_2\in\mathbb{R}^+$, then solve the following Linear Matrix Inequality (LMI):
$$AP+PA^T-\kappa_1BB^T+\kappa_2P\le 0\tag{4}$$
to get a $P>0$;
2) Let $K=-B^TP^{-1}$, $\Gamma=P^{-1}BB^TP^{-1}$ in (2) and (3);
3) Initialize $c_{ij}(0)=c_{ji}(0)\in\mathbb{R}^+$; select $\rho_{ij}=\rho_{ji},\theta,\eta\in\mathbb{R}^+$ and $\omega\in(0,1)$.

Now we are ready to formulate the main result of this paper.

Theorem 1: Suppose that the communication graph $\mathcal{G}$ is connected. Then, the consensus of MAS (1) can be solved with control law (2), triggering function (3), along with the parameters designed following Algorithm 1. Moreover, the adaptive coupling gains c_{ij} $(\forall i\in\mathcal{V},j\in\mathcal{N}_i)$ converge to some finite constants, and the closed-loop system does not exhibit Zeno behavior.

Proof. For brevity, denote for agent $i\in\mathcal{V}$ the global consensus error $e_i=x_i-\frac{1}{N}\sum_{j=1}^N x_j$, and $e_{ij}=e_i-e_j$. Then, it is clear that $e_{ij}\equiv x_{ij}$. Let $\boldsymbol{x}=\mathrm{col}(x_1,\cdots,x_N)$ and $\boldsymbol{e}=\mathrm{col}(e_1,\cdots,e_N)$.

Note that the gains c_{ij} satisfy a symmetric property, i.e., $c_{ij}\equiv c_{ji}$. In other words, we assign some independent adaptive coupling gains to the edges of the communication graph.

Based on (1) and (2), the closed-loop dynamics of x_i can be written as

$$\dot{x}_i=Ax_i+BK\sum_{j\in\mathcal{N}_i}c_{ij}\tilde{x}_{ij}.\tag{5}$$

Note that $\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}c_{ij}\tilde{x}_{ij}=0$. Then, the dynamics of e_i can be obtained as

$$\begin{aligned}\dot{e}_i=&Ae_i+BK\sum_{j\in\mathcal{N}_i}c_{ij}\tilde{x}_{ij}\\ =&Ae_i+BK\sum_{j\in\mathcal{N}_i}c_{ij}(x_{ij}+\epsilon_{ij}).\end{aligned}\tag{6}$$

It is clear that $\boldsymbol{e}=0$ amounts to $x_1=x_2=\cdots=x_N$, i.e., the consensus is achieved. In the following, we will show the asymptotic convergence of $\boldsymbol{e}$. To obtain this, consider the following candidate Lyapunov function:

$$V=\frac{1}{2}\boldsymbol{e}^T(\mathbf{I}_N\otimes P^{-1})\boldsymbol{e}+\frac{1-\omega}{16(1+\omega)}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\frac{(c_{ij}-\bar{c})^2}{\rho_{ij}}$$

where P is a solution of (4) and $\bar{c}\in\mathbb{R}^+$ remains to be decided.

The time derivative of V along system (6) is

$$\begin{aligned}\dot{V}=&\boldsymbol{e}^T(\mathbf{I}_N\otimes P^{-1}A)\boldsymbol{e}+\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}c_{ij}e_i^TP^{-1}BK(x_{ij}+\epsilon_{ij})\\ &+\frac{1-\omega}{8(1+\omega)}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}(c_{ij}-\bar{c})\tilde{x}_{ij}^T\Gamma\tilde{x}_{ij}.\end{aligned}\tag{7}$$

According to design of K and Γ and the symmetry of c_{ij}, we have

$$\begin{aligned}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}c_{ij}e_i^TP^{-1}BKx_{ij}=&-\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}c_{ij}e_i^T\Gamma e_{ij}\\ =&-\frac{1}{2}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}c_{ij}x_{ij}^T\Gamma x_{ij},\end{aligned}\tag{8}$$

and

$$\begin{aligned}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}c_{ij}e_i^TP^{-1}BK\epsilon_{ij}=&-\frac{1}{2}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}c_{ij}e_{ij}^T\Gamma\epsilon_{ij}\\ \le&\frac{1}{4}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}c_{ij}(x_{ij}^T\Gamma x_{ij}+\epsilon_{ij}\Gamma\epsilon_{ij}).\end{aligned}\tag{9}$$

Moreover,

$$\frac{1-\omega}{8(1+\omega)}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}c_{ij}\tilde{x}_{ij}^T\Gamma\tilde{x}_{ij} \leq\frac{1-\omega}{4(1+\omega)}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}c_{ij}(x_{ij}^T\Gamma x_{ij}+\epsilon_{ij}\Gamma\epsilon_{ij}) \quad (10)$$

where we have used the fact that $(a+b)^2 \leq 2a^2+2b^2$ to get the inequality. It follows from (7)-(10) and some direct manipulations that

$$\begin{aligned}\dot{V} \leq& \boldsymbol{e}^T(\mathbf{I}_N\otimes P^{-1}A)\boldsymbol{e}-\frac{\omega}{2(1+\omega)}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}c_{ij}x_{ij}^T\Gamma x_{ij}\\ &+\frac{1}{2(1+\omega)}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}c_{ij}\epsilon_{ij}\Gamma\epsilon_{ij}\\ &-\frac{\bar{c}(1-\omega)}{8(1+\omega)}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\tilde{x}_{ij}^T\Gamma\tilde{x}_{ij}. \end{aligned}\quad (11)$$

For each $j\in\mathcal{N}_i$, the Young's inequality guarantees that

$$\begin{aligned}x_{ij}^T\Gamma x_{ij}&=\tilde{x}_{ij}^T\Gamma\tilde{x}_{ij}+\epsilon_{ij}\Gamma\epsilon_{ij}-2\tilde{x}_{ij}^T\Gamma\epsilon_{ij}\\ &\leq\frac{1+\gamma}{\gamma}\tilde{x}_{ij}^T\Gamma\tilde{x}_{ij}+(1+\gamma)\epsilon_{ij}\Gamma\epsilon_{ij}\end{aligned}\quad (12)$$

where $\gamma\in\mathbb{R}^+$ whose value will be specified in a while. Furthermore, governed by the event-triggered sampling mechanism, we always have $f_{ij}\leq 0$ in (3), i.e.,

$$\epsilon_{ij}^T\Gamma\epsilon_{ij}\leq\omega x_{ij}^T\Gamma x_{ij}+\frac{\theta}{c_{ij}}\mathrm{e}^{-\eta t}, \quad (13)$$

which, combined with (12), gives

$$-\tilde{x}_{ij}^T\Gamma\tilde{x}_{ij}\leq-\frac{\gamma(1-\omega-\omega\gamma)}{1+\gamma}x_{ij}^T\Gamma x_{ij}+\frac{\gamma\theta}{c_{ij}}\mathrm{e}^{-\eta t}. \quad (14)$$

Besides, (13) also guarantees that

$$\begin{aligned}&\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}c_{ij}\epsilon_{ij}\Gamma\epsilon_{ij}-\omega\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}c_{ij}x_{ij}^T\Gamma x_{ij}\\ &\quad\leq\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\mathrm{e}^{-\eta t}\leq\frac{\theta N(N-1)}{2}\mathrm{e}^{-\eta t}.\end{aligned}\quad (15)$$

Thus, it follows from (11), (14) and (15) that

$$\begin{aligned}\dot{V}\leq&\boldsymbol{e}^T(\mathbf{I}_N\otimes P^{-1}A)\boldsymbol{e}+\frac{\theta N(N-1)}{4(1+\omega)}\mathrm{e}^{-\eta t}\\ &-\frac{\bar{c}\gamma(1-\omega)(1-\omega-\omega\gamma)}{8(1+\omega)(1+\gamma)}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}x_{ij}^T\Gamma x_{ij}\\ &+\frac{\bar{c}\gamma\theta(1-\omega)}{8(1+\omega)}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\frac{\mathrm{e}^{-\eta t}}{c_{ij}}.\end{aligned}$$

Note that

$$\begin{aligned}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}x_{ij}^T\Gamma x_{ij}&=2\boldsymbol{x}^T(EE^T\otimes\Gamma)\boldsymbol{x}\\ &=2\boldsymbol{x}^T(\mathcal{L}\otimes\Gamma)\boldsymbol{x}=2\boldsymbol{e}^T(\mathcal{L}\otimes\Gamma)\boldsymbol{e},\end{aligned}$$

which leads to

$$\dot{V}\leq\boldsymbol{e}^T(\mathbf{I}_N\otimes P^{-1}A)\boldsymbol{e}-\bar{c}\mu\boldsymbol{e}^T(\mathcal{L}\otimes\Gamma)\boldsymbol{e}+\nu\mathrm{e}^{-\eta t}$$

where $\mu=\frac{\gamma(1-\omega)(1-\omega-\omega\gamma)}{4(1+\omega)(1+\gamma)}$, and $\nu=\frac{\bar{c}\gamma\theta N(N-1)(1-\omega)}{16(1+\omega)\min\{c_{ij}(0)\}}+\frac{\theta N(N-1)}{4(1+\omega)}$. Let $\bar{\boldsymbol{e}}=(\mathbf{I}_N\otimes P^{-1})\boldsymbol{e}$, it follows that

$$\begin{aligned}\dot{V}\leq&\frac{1}{2}\bar{\boldsymbol{e}}^T\big(\mathbf{I}_N\otimes(AP+PA^T)-2\bar{c}\mu\mathcal{L}\otimes BB^T\big)\bar{\boldsymbol{e}}+\nu\mathrm{e}^{-\eta t}\\ \leq&\frac{1}{2}\bar{\boldsymbol{e}}^T\big(\mathbf{I}_N\otimes(AP+PA^T-2\bar{c}\mu\lambda_2(\mathcal{L})BB^T)\big)\bar{\boldsymbol{e}}+\nu\mathrm{e}^{-\eta t}.\end{aligned}$$

Now, let us first choose $\gamma<\frac{1}{\omega}-1$, which implies $\mu>0$ (recall that $\omega\in(0,1)$ in Algorithm 1). Then, choose $\bar{c}$ such that $\bar{c}>\frac{\kappa_1}{2\mu\lambda_2(\mathcal{L})}$ where κ_1 is defined as in (4). This gives

$$\begin{aligned}\dot{V}\leq&-\frac{\kappa_2}{2}\bar{\boldsymbol{e}}^T(\mathbf{I}_N\otimes P)\bar{\boldsymbol{e}}+\nu\mathrm{e}^{-\eta t}\\ =&-\frac{\kappa_2}{2}\boldsymbol{e}^T(\mathbf{I}_N\otimes P^{-1})\boldsymbol{e}+\nu\mathrm{e}^{-\eta t}.\end{aligned}\quad (16)$$

Thus,

$$\begin{aligned}0\leq V(t)&=V(0)+\int_0^t\dot{V}(\tau)d\tau\\ &\leq V(0)+\int_0^t\nu\mathrm{e}^{-\eta\tau}d\tau\leq V(0)+\frac{\nu}{\eta},\end{aligned}$$

which implies that $V(t)$ is bounded, which further guarantees that $\boldsymbol{e}(t)$ (and thus $\boldsymbol{x}(t)$) and $c_{ij}(t)$ and $\epsilon_{ij}(t)$, for all edges, are bounded. Since the coupling gains c_{ij} are monotonically increasing by construction, each of them would converge to some finite constant. Note by (6) that $\dot{\boldsymbol{e}}$ is also bounded, which indicates that the function $\boldsymbol{e}^T(\mathbf{I}_N\otimes P^{-1})\boldsymbol{e}$ is uniformly continuous. Furthermore, by (16), one has

$$\int_0^\infty\boldsymbol{e}(\tau)^T(\mathbf{I}_N\otimes P^{-1})\boldsymbol{e}(\tau)d\tau\leq\frac{2\nu}{\kappa_2\eta}+\frac{V(0)-V(\infty)}{\kappa_2}<\infty. \quad (17)$$

According to the well known Barbalat's Lemma [18], we have $\lim_{t\to\infty}\boldsymbol{e}^T(t)(\mathbf{I}_N\otimes P^{-1})\boldsymbol{e}(t)=0$. Then, we can conclude that $\lim_{t\to\infty}\boldsymbol{e}(t)=0$, i.e., the MAS (1) reaches a consensus.

Next, we show that the closed-loop system is Zeno-free. To this end, let us denote $\epsilon_{ij}^\Gamma(t)=\epsilon_{ij}(t)^T\Gamma\epsilon_{ij}(t)$ for $(i,j)\in\mathcal{E}$ and $t\in[t_k^{ij},t_{k+1}^{ij})$. By definition,

$$\begin{aligned}\frac{d\epsilon_{ij}^\Gamma}{dt}&=2\epsilon_{ij}^T\Gamma(\dot{x}_i-\dot{x}_j)\\ &=2\epsilon_{ij}^T\Gamma(Ax_{ij}+B(u_i-u_j)).\end{aligned}$$

Since $\boldsymbol{x}(t)$, $\epsilon_{ij}(t)$ and $c_{ij}(t)$ are bounded, we know that u_i are bounded. If we denote $\chi_{ij}=\sup_{t\geq0}|2\epsilon_{ij}^T\Gamma(Ax_{ij}+B(u_i-u_j))|$, then $0<\chi_{ij}<\infty$. As $\epsilon_{ij}^\Gamma(t_k^{ij})=0$ and $\frac{d\epsilon_{ij}^\Gamma(t)}{dt}\leq\chi_{ij}$, it follows that $\epsilon_{ij}^\Gamma(t)\leq\chi_{ij}(t-t_k^{ij})$, $t\in[t_k^{ij},t_{k+1}^{ij})$. Note by (3) that $\epsilon_{ij}^\Gamma(t_{k+1}^{ij})=\omega x_{ij}^T\Gamma x_{ij}|_{t=t_{k+1}^{ij}}-\frac{\theta}{c_{ij}}\mathrm{e}^{-\eta t_{k+1}^{ij}}$, then

$$\frac{\theta}{c_{ij}}\mathrm{e}^{-\eta t_{k+1}^{ij}}\leq\epsilon_{ij}^\Gamma(t_{k+1}^{ij})\leq\chi_{ij}(t_{k+1}^{ij}-t_k^{ij}), \quad (18)$$

that is $t_{k+1}^{ij}-t_k^{ij}\geq\frac{\theta}{\chi_{ij}c_{ij}}\mathrm{e}^{-\eta t_{k+1}^{ij}}>0$ for any finite horizon. Moreover, it is not hard to conclude that $t_k^{ij}\to\infty$ as $t\to\infty$

by contradiction [16]. Then, no Zeno behavior would exist. This completes the proof. ■

As compared to related methods available in the literature, let us explain the contributions of the proposed method in some remarks.

Remark 2: In the most recent edge-based event-triggered control method proposed in [15], two triggering functions are designed for each edge. While in the proposed method, only one triggering function is designed for each edge, which is more convenient and intuitive.

Remark 3: As compared with the node-based strategy in [16], the proposed edge-based method can be applied to the case of switching connected graphs. This conclusion follows immediately when noticing that the Lyapunov function in the proof is not related to the graph Laplacian matrix $\mathcal{L}$. We summarize this point into a corollary below.

Corollary 1: Suppose that the communication graph is switching over several connected graphs. Let $\mathcal{N}_i^t$ be the neighbors of i at time t. Then, the consensus of MAS (1) can be solved with control law

$$\begin{aligned} u_i &= K \sum_{j\in\mathcal{N}_i^t} c_{ij}\tilde{x}_{ij} \\ \dot{c}_{ij} &= \rho_{ij}\tilde{x}_{ij}^T\Gamma\tilde{x}_{ij}, j\in\mathcal{N}_i^t, \end{aligned} \tag{19}$$

triggering function (3), along with the parameters designed following Algorithm 1. Moreover, the adaptive coupling gains c_{ij} ($\forall i\in\mathcal{V}, j\in\mathcal{N}_i^t$) converge to some finite constants, and the closed-loop system does not exhibit Zeno behavior.

IV. An Illustrative Example

In this section, we propose a numerical example to validate our method. Consider a MAS consisting of 6 second order oscillators where

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

In view of Corollary 1, the communication graph is assumed randomly switching between a Star graph (with node 1 being the center) and a Ring graph.

Following Algorithm 1, let $\kappa_1 = \kappa_2 = 1$, and solve the LMI (4). Then, the obtained feedback gain matrices are $K = (-6, -6)$ and $\Gamma = \begin{pmatrix} 36 & 36 \\ 36 & 36 \end{pmatrix}$. We initialize $c_{ij}(0) = 1$ for any edge and select $\rho_{ij} = 0.5$. For the triggering functions, we choose $\theta = \eta = 0.5$, $\omega = 0.9$.

The initial states of the agents are randomly chosen. After the implementation of the distributed event-triggered adaptive consensus controller (2), the states of the agents are provided in Fig. 1. The global consensus error $e_c(t) \triangleq \sqrt{\frac{1}{6}\sum_{i=1}^{6}\|e_i(t)\|^2}$ in the MAS converges to zero, as shown in Fig. 2. The triggering instants for each edge are shown in Fig. 3, while the adaptive coupling gains for each of them are shown in Fig. 4. Fig. 5 shows the communication topology switching indicator. Finally, the control inputs for the agents are provided in Fig. 6. It is clear that continuously samplings are avoided, meanwhile the adaptive gains converge. The theoretical results are well supported.

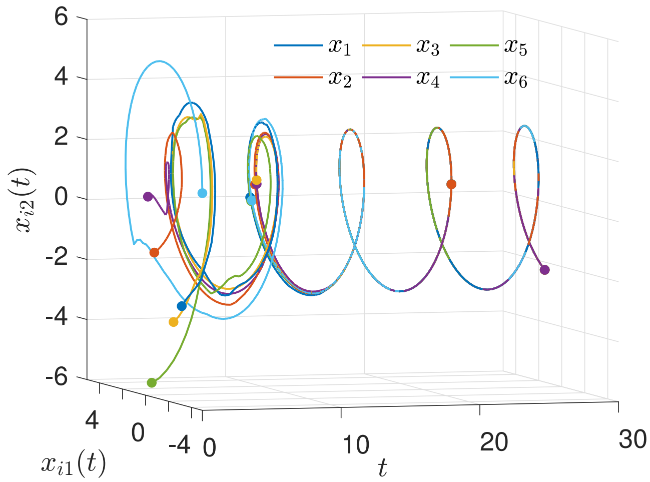

Fig. 1: The states $x_i(t)$ of the agents.

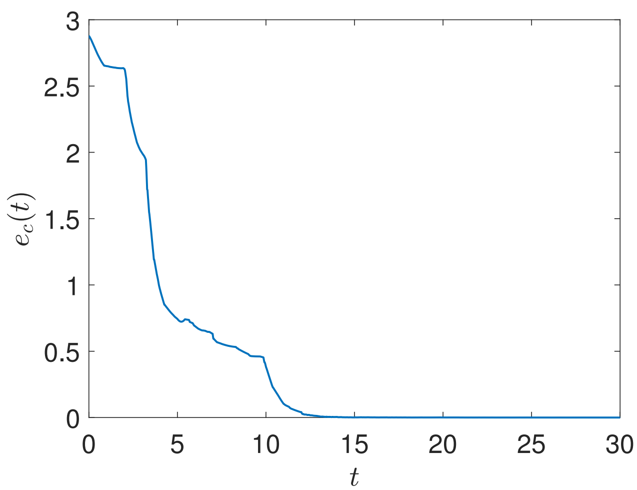

Fig. 2: The global consensus error $e_c(t)$ in the MAS.

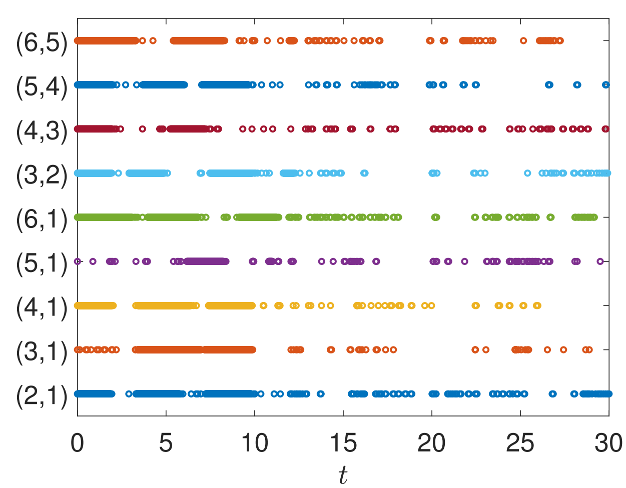

Fig. 3: The triggering instants of all edges (i, j).

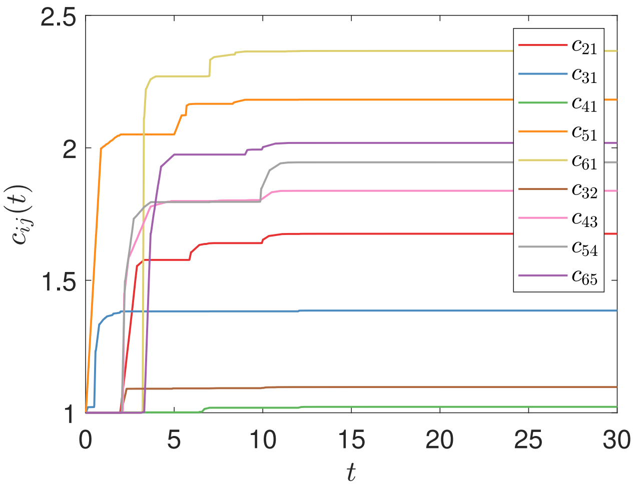

Fig. 4: The adaptive coupling gains $c_{ij}(t)$ of all edges (i, j).

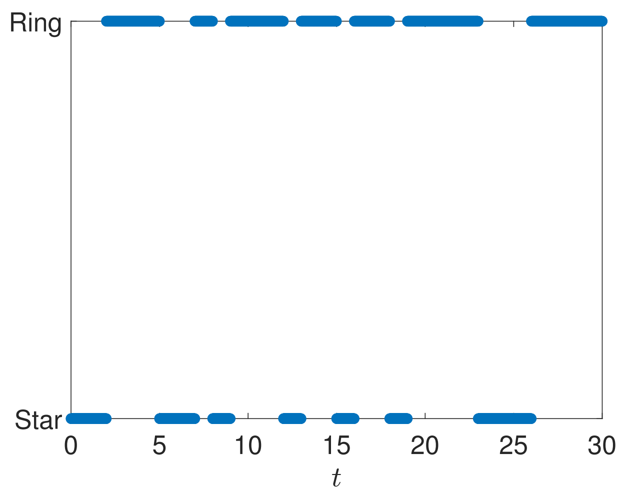

Fig. 5: The communication topology switching indicator.

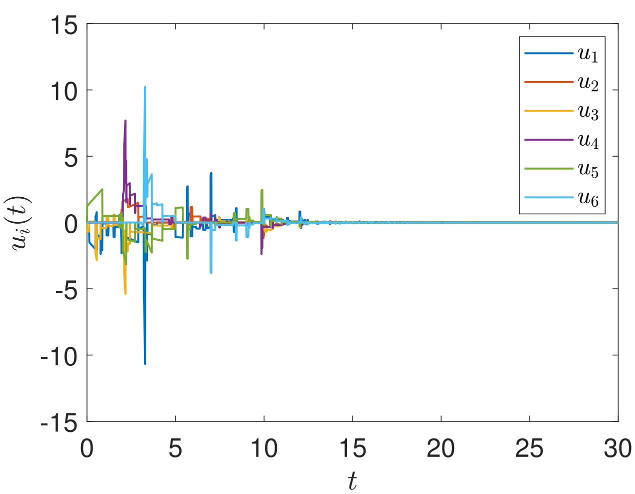

Fig. 6: The control inputs $u_i(t)$ of the agents.

V. Conclusion

This brief addressed distributed adaptive consensus of linear multiagent systems based on event-triggered sampling. A novel class of triggering functions was designed in terms of the edges in the network to determine the sampling instants of pair-wise relative consensus errors. Meanwhile, adaptive coupling gains were designed, consistently, in terms of the edges in the network. The proposed method is fully distributed without requiring any global information. Future works may involve uncertain agents as in [16], and/or directed communication topologies using the method of [19].

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