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Distributed Adaptive Consensus Disturbance Rejection: a Directed-spanning-tree Perspective

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Abstract: In this paper, we revisit the problem of consensus disturbance rejection for multiagent systems over a digraph, but from a different perspective, i.e., the perspective of a directed spanning tree (DST). When the minimum nonzero real part of the Laplacian eigenvalues is available, we reproduce the sufficient lower bound for a static homogeneous coupling gain in the literature, by exploring a DST structure of the digraph. The major novelty arises when it is shown that by adaptively tuning the coupling gains along a DST, consensus disturbance rejection can be achieved when the above eigenvalue information is not available. Numerical examples on networks of second-order oscillators and UAVs are included to validate the theoretical results.

Key Words: Distributed adaptive control, Consensus, Disturbance rejection, Directed spanning tree

1 Introduction

Cooperative consensus of multiagent systems appears in a variety of scenarios such as decision making [1], distributed optimization [2], robots [3], etc. According to whether there is a reference agent, the consensus problems can be roughly divided into leaderless consensus [4, 5] and leader-follower consensus [6, 7]. Note that a critical point in the leaderless setting is that the agents reach consensus by purely self-organizing their behaviors.

Disturbance rejection (also known as disturbance attenuation, regulation, anti-disturbance, etc.), on the other hand, is a longstanding problem in control system design [8–10]. The problem is challenging since external disturbances are either too expensive or even impossible to be directly measured, which makes a feedforward compensation strategy not applicable. In this scenario, an intuitive idea is to estimate the disturbance and cancel its influence. Such an idea has evolved into the disturbance observer based control, see e.g., [11, 12] for related results and applications.

When considering disturbances rejection problems for multiagent systems, some results have been reported recently [13–15]. The work of [13, 14] addressed the disturbance by means of discontinuous control, which may induce extra oscillations. Using the pure gain feedback control strategy, [15] proposed a disturbance observer based design for consensus disturbance rejection. It was shown in [15] that when the communication graph among the agents contains a directed spanning tree (DST), consensus disturbance rejection can be attained, provided that the static coupling gain exceeds a lower bound. Such a lower bound relies on the minimum nonzero real part of the Laplacian eigenvalues, which may not be available in real networks, especially when the network size is large.

Motivated by the above discussions, we reconsider the consensus disturbance rejection for linear multiagent systems, but from a directed spanning tree perspective. The existence of a DST structure is widely known as an assumption in cooperative control of multiagent systems, but is rarely explored in detail. A major contribution of this paper is to propose a DST-based adaptive consensus disturbance rejection scheme, which does not rely on the global information of the Laplacian eigenvalues (Theorem 1). The framework covers existing results as a special case (Lemma 3).

Notations: Denote $\mathbb{R}$ as the real space, and $\mathcal{I}_N$ as the set $\{1, 2, \cdots, N\}$. For a matrix A, let null(A) be its zero space; if A is square, let $\lambda(A)$ be its eigenvalue and $\Re(\lambda(A))$ be the real part. For a vector a, span(a) is the real space spanned by a, i.e., $\{\kappa a | \forall \kappa \in \mathbb{R}\}$. Let $\otimes$ be the Kronecker product, and $*$ be the complex conjugate. The rest notations are standard.

2 Problem statement and technical lemmas

Consider a class of multiagent systems with dynamics

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t) + D\omega_i(t), \ \ i \in \mathcal{I}_N, \tag{1}$$

where $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^m$ are, respectively, the state and control input of agent i. Agent i is disturbed by unmeasurable $\omega_i \in \mathbb{R}^s$, which is generated by an exosystem

$$\dot{\omega}_i(t) = E\omega_i(t), \ \ i \in \mathcal{I}_N. \tag{2}$$

In (1)-(2), A, B, D and E are known constant matrices with compatible dimensions. Note that the exosystem dynamics is identical for all agents [15, 16], which is reasonable as the agents are usually supposed to work under the same conditions. We make the following assumptions on the system dynamics [15].

Assumption 1 *There exist a matrix F such that $D = BF$.*

Assumption 2 *The eigenvalues of the matrix E are simple with zero real parts.*

Assumption 3 *The pair (A, B) is stabilizable; the pair (E, D) is observable.*

Remark 1 *Assumption 1 is known as the matching condition. Assumption 2 guarantees that the disturbances are non-*

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vanishing sinusoidal functions or constants. The observability of (E, D) is reasonable since any unobservable components would have no impact on the agents.

Lemma 1 ([15]) *Under the observability of (E, D) of Assumption 3, the pair (A_H, H) is also observable, where $A_H = \begin{pmatrix} A & D \\ 0_{s\times n} & E \end{pmatrix}$ and $H = (\boldsymbol{I}_n, 0_{n\times s})$.*

The communications among the agents are characterized by a *directed graph* (or simply *digraph*) $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \mathcal{I}_N$ is the node set, $\mathcal{E} = \{e_{ij}, i \neq j | i \to j\}$ is the edge set, and $\mathcal{A} = (a_{ij}) \in \mathbb{R}^{N\times N}$ is the adjacency matrix such that $a_{ij} > 0$ if $e_{ji} \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. The Laplacian matrix $\mathcal{L} = (l_{ij}) \in \mathbb{R}^{N\times N}$ associated with $\mathcal{G}$ consists of $l_{ij} = -a_{ij}$ for $i \neq j$, and $l_{ii} = \sum_{j=1}^{N} a_{ij}$. For $e_{ij} \in \mathcal{E}$, i is called an in-neighbor of j and j an out-neighbor of i in return: $i \in \mathcal{N}_{\text{in}}(j)$ and $j \in \mathcal{N}_{\text{out}}(i)$. A path is a sequence of edges connecting a pair of nodes, which respects the edge directions.

A directed spanning tree $\bar{\mathcal{G}}(\mathcal{V}, \bar{\mathcal{E}}, \bar{\mathcal{A}})$ of $\mathcal{G}$ is a subgraph which contains a root (has no in-neighbors), such that one can find a unique path from the root to every other node. Without loss of generality, we label the root as node 1. Following the notations in [2, 17, 18], let i_k denote the unique in-neighbor of node $k+1$ in $\bar{\mathcal{G}}$, $k \in \mathcal{I}_{N-1}$. Correspondingly, $\bar{\mathcal{L}}$ is the Laplacian matrix of $\bar{\mathcal{G}}$ and $\bar{\mathcal{N}}_{\text{out}}(i)$ is the set of out-neighbors of i in $\bar{\mathcal{G}}$.

Assumption 4 *The communication digraph $\mathcal{G}$ contains a directed spanning tree $\bar{\mathcal{G}}$.*

Let us construct two matrices based on the directed spanning tree $\bar{\mathcal{G}}$. Define $\Xi \in \mathbb{R}^{(N-1)\times N}$ as

$$\Xi_{kj} = \begin{cases} -1, & \text{if } j = k+1, \\ 1, & \text{if } j = i_k, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

Define $\Pi \in \mathbb{R}^{(N-1)\times(N-1)} := \tilde{\Pi} + \bar{\Pi}$ with

$$\Pi_{kj} = \underbrace{\sum_{c\in\bar{\mathcal{V}}_{j+1}} (\tilde{\mathcal{L}}_{k+1,c} - \tilde{\mathcal{L}}_{i_k,c})}_{\tilde{\Pi}_{kj}} + \underbrace{\sum_{c\in\bar{\mathcal{V}}_{j+1}} (\bar{\mathcal{L}}_{k+1,c} - \bar{\mathcal{L}}_{i_k,c})}_{\bar{\Pi}_{kj}}, \tag{4}$$

where $\tilde{\mathcal{L}} = \mathcal{L} - \bar{\mathcal{L}}$, and $\bar{\mathcal{V}}_{j+1}$ represents the vertex set of the subtree of $\bar{\mathcal{G}}$ rooting at node $j+1$.

Lemma 2 ([2]) *Under Assumption 4, the following statements hold for $\mathcal{L}$ (of $\mathcal{G}$), and Ξ, Π defined above:*

1) $0 = \lambda_1(\mathcal{L}) < \Re(\lambda_2(\mathcal{L})) \leq \Re(\lambda_3(\mathcal{L})) \leq \cdots \leq \Re(\lambda_N(\mathcal{L}))$. *Moreover, null*$(\mathcal{L})$ = *span*$(\boldsymbol{1}_N)$.
2) $\Xi\mathcal{L} = \Pi\Xi$. *Moreover, null*(Ξ) = *span*$(\boldsymbol{1}_N)$.
3) $\lambda_i(\Pi) = \lambda_{i+1}(\mathcal{L})$, $i \in \mathcal{I}_{N-1}$.
4) $\bar{\Pi}$ *can be explicitly written as*

$$\bar{\Pi}_{kj} = \begin{cases} \bar{a}_{j+1,i_j}, & \text{if } j = k, \\ -\bar{a}_{j+1,i_j}, & \text{if } j = i_k - 1, \\ 0, & \text{otherwise.} \end{cases}$$

The consensus disturbance rejection problem is to drive the agents to consensus despite the effect of the disturbances, i.e., $\lim_{t\to\infty} \big(x_i(t) - x_j(t)\big) = 0, \forall i, j \in \mathcal{I}_N$.

3 Main results

In this section, we provide a DST-based adaptive consensus disturbance rejection scheme. To start with, we consider the non-adaptive (static) case studied in [15, Section IV], i.e., under the assumption that the information of $\Re(\lambda_2(\mathcal{L}))$ is available.

3.1 Static coupling case: a DST perspective

The control input for agent i is designed based on a state observer and a disturbance observer. Specifically,

$$u_i = -K\chi_i - Fz_i \tag{5a}$$

$$\dot{\chi}_i = (A - BK)\chi_i + c\Gamma_x \sum_{j=1}^{N} a_{ij}(\rho_i - \rho_j) \tag{5b}$$

$$\dot{z}_i = Ez_i + c\Gamma_\omega \sum_{j=1}^{N} a_{ij}(\rho_i - \rho_j) \tag{5c}$$

where $\rho_i = x_i - \chi_i$; K, Γ_x and Γ_ω are gain matrices to be determined and $c \in \mathbb{R}^+$ represent the static coupling strength among the agents. Note that only relative state information is involved in the scheme, which suits the case when absolute state measurement is not available.

Since (A, B) is stabilizable, there exists a $P > 0$ such that

$$AP + PA^T - 2BB^T < 0. \tag{6}$$

Moreover, since (A_H, H) is observable by Lemma 1, there exists a $Q > 0$ such that

$$QA_H + A_H^T Q - 2H^T H < 0. \tag{7}$$

We have the following lemma.

Lemma 3 *Under Assumptions 1-4, the consensus disturbance rejection problem of the multiagent system (1) can be solved by the static scheme (5). The parameters are designed as $K = B^T P^{-1}$, $\Gamma := (\Gamma_x^T, \Gamma_\omega^T)^T = Q^{-1}H^T$, and $c \geq \frac{1}{\Re(\lambda_2(\mathcal{L}))}$.*

Proof. The lemma is straightforward by [15, Theorem 4]. However, let us sketch an alternative proof below from a DST perspective, which also helps for a better understanding of the DST-based adaptive scheme later on.

Denote $e_i = \begin{pmatrix} x_i - \chi_i \\ \omega_i - z_i \end{pmatrix}$ as the composite observer error system. It is clear that $\rho_i = He_i$. For the ease of analysis, let us stack the vectors as $e = (e_1^T, \cdots, e_N^T)$. Based on (1), (2) and (5), the dynamics of e can be obtained as

$$\dot{e} = (\mathbf{I}_N \otimes A_H - c\mathcal{L} \otimes \Gamma H)e \tag{8}$$

Let us consider the transformation $\xi = (\Xi \otimes \mathbf{I}_{n+s})e$, where ξ is defined as in (3). The dynamics of ξ follows from (8) as

$$\dot{\xi} = (\mathbf{I}_{N-1} \otimes A_H - c\Pi \otimes \Gamma H)\xi \tag{9}$$

where we have used statement 2) of Lemma 2. We claim that ξ_i converge to zero by showing that the system matrix of ξ is Hurwitz. Note that any square matrix is unitarily similar to an upper triangular matrix with diagonal entries being its eigenvalues. When this fact is applied to Π and noticing statement 3) of Lemma 2, it is sufficient to show that the matrix

$$\begin{pmatrix} A_H - c\lambda_2(\mathcal{L})\Gamma H & \star & \star \\ & \ddots & \star \\ & & A_H - c\lambda_N(\mathcal{L})\Gamma H \end{pmatrix}$$

is Hurwitz, which is then equivalent to show that the diagonal blocks are Hurwitz. In fact, for any $i \in \{2, \cdots, N\}$,

$$\begin{aligned} & Q(A_H - c\lambda_i(\mathcal{L})\Gamma H) + (A_H - c\lambda_i(\mathcal{L})\Gamma H)^* Q \\ =& QA_H + A_H^T Q - c\lambda_i(\mathcal{L}) H^T H - c\lambda_i^*(\mathcal{L}) H^T H \\ =& QA_H + A_H^T Q - 2c\Re(\lambda_i(\mathcal{L})) H^T H \\ \leq& QA_H + A_H^T Q - 2H^T H < 0. \end{aligned} \tag{10}$$

where we have used $\Gamma = Q^{-1}H^T$, $c \geq \frac{1}{\Re(\lambda_2(\mathcal{L}))}$, and the LMI (7). The above implies that $A_H - c\lambda_i(\mathcal{L})\Gamma H$ for $i \in \{2, \cdots, N\}$ are indeed Hurwitz. This guarantees that ξ converges to zero.

On the other hand, under Assumption 1, we have

$$\begin{aligned} \dot{x}_i &= Ax_i - BK\chi_i - BFz_i + D\omega_i \\ &= (A - BK)x_i + B(K, F)e_i \end{aligned} \tag{11}$$

for each agent i. If we stack the state and consider again the transformation $\delta = (\Xi \otimes \mathbf{I}_n)x$, the dynamics of δ can be obtained as

$$\dot{\delta} = \big(\mathbf{I}_{N-1} \otimes (A - BK)\big)\delta + \big(\mathbf{I}_{N-1} \otimes B(K, F)\big)\xi. \tag{12}$$

With $K = B^T P^{-1}$, $A - BK$ is Hurwitz. Then, as ξ converges to zero, it is clear that δ converges to zero. Since $\text{null}(\Xi) = \text{span}(\mathbf{1}_N)$, the convergence of δ to zero is equivalent to the consensus of x_i, i.e., the consensus disturbance problem is solved. The proof is completed. ∎

3.2 Adaptive coupling case: a DST perspective

We move on to the adaptive case when the information of $\Re(\lambda_2(\mathcal{L}))$ is not available. The idea is to promote the consensus over ρ_i on the basis of the static scheme (5), by adaptively tuning the coupling gains along a DST. Specifically, the controller for agent i is proposed as

$$u_i = -K\chi_i - Fz_i \tag{13a}$$

$$\dot{\chi}_i = (A - BK)\chi_i + \Gamma_x \sum_{j=1}^{N} c_{ij}a_{ij}(\rho_i - \rho_j) \tag{13b}$$

$$\dot{z}_i = Ez_i + \Gamma_\omega \sum_{j=1}^{N} c_{ij}a_{ij}(\rho_i - \rho_j) \tag{13c}$$

$$\dot{c}_{ij} = \begin{cases} \gamma\Big((\rho_{i_k} - \rho_{k+1}) - \sum\limits_{j \in \bar{\mathcal{N}}_{\text{out}}(k+1)} (\rho_{k+1} \\ \quad -\rho_j)\Big)^T (\rho_{i_k} - \rho_{k+1}) \triangleq \dot{\bar{c}}_{k+1,i_k}, & \text{if } e_{ji} \in \bar{\mathcal{E}} \\ 0, & \text{if } e_{ji} \in \mathcal{E} \setminus \bar{\mathcal{E}} \end{cases} \tag{13d}$$

where $\gamma \in \mathbb{R}^+$ is a constant gain. By construction, the coupling gain between agent i and its in-neighbor j is updating only when the edge e_{ji} appears in $\bar{\mathcal{G}}$ (i.e., $j = i_k$ and $i = k + 1$ for some $k \in \mathcal{I}_{N-1}$).

The convergence result of (13) is summarized below.

Theorem 1 *Under Assumptions 1-4, the consensus disturbance rejection problem of the multiagent system (1) can be solved by the adaptive scheme (13). The parameters are designed as $K = B^T P^{-1}$, $\Gamma = Q^{-1}H^T$, and $\gamma \in \mathbb{R}^+$. Moreover, the gains $\bar{c}_{k+1,i_k}$, $k \in \mathcal{I}_{N-1}$, in $\bar{\mathcal{G}}$ converge to some finite constant values.*

Proof. Define $\mathcal{L}^c$ as the gain-dependent Laplacian matrix as

$$\mathcal{L}^c_{ij} = -c_{ij}a_{ij},\ i \neq j;$$
$$\mathcal{L}^c_{ii} = \sum_{j=1, j\neq i}^{N} c_{ij}a_{ij},\ i \in \mathcal{I}_N.$$

Then, following the notations in the previous subsection, we have that the dynamics of e is

$$\dot{e} = (\mathbf{I}_N \otimes A_H - \mathcal{L}^c \otimes \Gamma H)e. \tag{14}$$

Similar to (9), the dynamics of ξ is

$$\dot{\xi} = (\mathbf{I}_{N-1} \otimes A_H - \Pi^c \otimes \Gamma H)\xi \tag{15}$$

where Π^c is defined as in (4) based on the DST $\bar{\mathcal{G}}$ and the gain-dependent Laplacian matrix. More specifically, $\Pi^c = \tilde{\Pi}^c + \bar{\Pi}^c$ contains the fixed matrix $\tilde{\Pi}^a$ (note that $\dot{c}_{ij} = 0$ if $e_{ji} \in \mathcal{E} \setminus \bar{\mathcal{E}}$), and the time-varying matrix

$$\bar{\Pi}^c_{kj} = \begin{cases} \bar{c}_{j+1,i_j}a_{j+1,i_j}, & \text{if } j = k, \\ -\bar{c}_{j+1,i_j}a_{j+1,i_j}, & \text{if } j = i_k - 1, \\ 0, & \text{otherwise}. \end{cases} \tag{16}$$

Note that in component-wise form, $\xi_k = e_{i_k} - e_{k+1}$, $k \in \mathcal{I}_{N-1}$. Then, we can rewrite $\dot{\bar{c}}_{k+1,i_k}$ in (13d) as

$$\dot{\bar{c}}_{k+1,i_k} = \gamma(H\xi_k - \sum_{j+1 \in \bar{\mathcal{N}}_{\text{out}}(k+1)} H\xi_j)^T H\xi_k. \tag{17}$$

As before, we show that ξ converges to zero. Consider the candidate Lyapunov function

$$\begin{aligned} V =& \frac{1}{2}\xi^T(\mathbf{I}_{N-1} \otimes Q)\xi \\ &+ \sum_{k=1}^{N-1} \frac{a_{k+1,i_k}}{2\gamma}(\bar{c}_{k+1,i_k}(t) - \phi_{k+1,i_k})^2 \end{aligned}$$

where Q is a solution to (7) and $\phi_{k+1,i_k} \in \mathbb{R}^+$, $k \in \mathcal{I}_{N-1}$, are constants to be decided later. The derivative of V along (15) and (17) can be obtained as

$$\begin{aligned} \dot{V} =& \xi^T(\mathbf{I}_{N-1} \otimes QA_H - \Pi^c \otimes Q\Gamma H)\xi \\ &+ \sum_{k=1}^{N-1} a_{k+1,i_k}(\bar{c}_{k+1,i_k} - \phi_{k+1,i_k})\Big(H\xi_k \\ &- \sum_{j+1 \in \bar{\mathcal{N}}_{\text{out}}(k+1)} H\xi_j)^T H\xi_k. \end{aligned} \tag{18}$$

From (16), one has

$$\begin{aligned} & \sum_{k=1}^{N-1} a_{k+1,i_k}\bar{c}_{k+1,i_k}\Big(H\xi_k - \sum_{j+1 \in \bar{\mathcal{N}}_{\text{out}}(k+1)} H\xi_j)^T H\xi_k \\ =& \sum_{k=1}^{N-1} (\bar{\Pi}^c_{kk}H\xi_k + \sum_{j=1, j\neq k}^{N-1} \bar{\Pi}^c_{jk}H\xi_j)^T H\xi_k \\ =& \sum_{k=1}^{N-1}\sum_{j=1}^{N-1} \bar{\Pi}^c_{jk}\xi_j^T H^T H\xi_k = \xi^T(\bar{\Pi}^c \otimes H^T H)\xi. \end{aligned} \tag{19}$$

Let us define $\Phi \in \mathbb{R}^{(N-1)\times(N-1)}$ as

$$\Phi_{kj} = \begin{cases} \phi_{j+1,i_j} a_{j+1,i_j}, & \text{if } j = k, \\ -\phi_{j+1,i_j} a_{j+1,i_j}, & \text{if } j = i_k - 1, \\ 0, & \text{otherwise}. \end{cases} \tag{20}$$

Then, it follows from (18)-(20) and $\Gamma = Q^{-1}H^T$ that

$$\begin{aligned} \dot{V} =& \xi^T(\mathbf{I}_{N-1} \otimes QA_H - \Pi^c \otimes H^T H)\xi \\ &+ \xi^T\big((\bar{\Pi}^c - \Phi) \otimes H^T H\big)\xi \\ =& \xi^T(\mathbf{I}_{N-1} \otimes QA_H)\xi - \xi^T\big((\tilde{\Pi}^c + \Phi) \otimes H^T H\big)\xi \\ =& \frac{1}{2}\xi^T\Big(\mathbf{I}_{N-1} \otimes (QA_H + A_H^T Q) \\ &- (\tilde{\Pi}^c + (\tilde{\Pi}^c)^T + \Phi + \Phi^T) \otimes H^T H\Big)\xi. \end{aligned} \tag{21}$$

Following similar procedures as in [2, 17, 18], one can show that $\Phi + \Phi^T > 0$ by selecting $\phi_{k+1,i_k} > \frac{\sum_{j=2}^{k} \phi_{j,i_{j-1}}^2 a_{j,i_{j-1}}^2}{2a_{k+1,i_k}\lambda_{\min}(\Omega_{k-1})}$, where $\Omega_1 = \begin{pmatrix} 2\phi_{2,i_1}a_{2,i_1} \end{pmatrix}$, and $\Omega_k = \begin{pmatrix} \Omega_{k-1} & \varphi_k \\ \varphi_k^T & 2\phi_{k+1,i_k}a_{k+1,i_k} \end{pmatrix}$ with $\varphi_k = (\phi_{k1}a_{k1}, \phi_{k2}a_{k2}, \cdots, \phi_{k,k-1}a_{k,k-1})^T$, $k = 2, \cdots, N-1$. Furthermore, since the matrix $\tilde{\Pi}^c$ is fixed, one can always select larger ϕ_{k+1,i_k}, $k \in \mathcal{I}_{N-1}$, such that $\lambda_{\min}(\tilde{\Pi}^c + (\tilde{\Pi}^c)^T + \Phi + \Phi^T) \geq 2$. Then, it follows from (21) that

$$\dot{V} \leq \frac{1}{2}\xi^T\big(\mathbf{I}_{N-1} \otimes (QA_H + A_H^T Q - 2H^T H)\big)\xi.$$

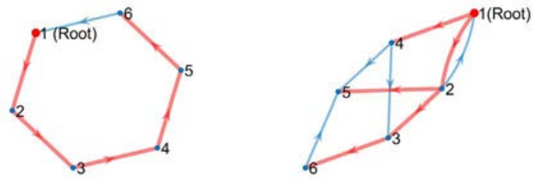
Based on LMI (7), we have $\dot{V} \leq 0$, where the equality holds only if $\xi = 0$. By LaSalle's invariance principle [19], it follows that ξ converges to zero. As a consequence, x_i reach consensus following similar analysis as in Lemma 3, and the adaptive gains $\bar{c}_{k+1,i_k}$ in the DST converge to some finite constants (since e_i reach consensus as ξ converges to zero). The proof is completed. ■

Remark 2 *The proposed DST-based adaptive consensus disturbance rejection scheme (13) is a natural extension of [15, Section V], from undirected graph to digraph with a DST. The design and analysis is essentially more complex due to the communication graph being asymmetric.*

Remark 3 *In [20], a class of node-based adaptive consensus disturbance rejection scheme was proposed for a leader-follower network with exact disturbance observers, and the idea was generalized to switching digraphs in [21]. Some differences of our method are worth to remark: first, the disturbance observer follows a similar philosophy as[15] in the sense that the disturbances that does not influence the consensus manifold does not need to be rejected; second, a leaderless network is considered where a reference agent does not exist; third, the proposed DST-based adaptive method is essentially different from the node-based one where the adaptive gains are designed for each agent (see [2, 18] for detailed comparisons).*

4 Numerical examples

In this section, we give two examples to validate the proposed DST-based adaptive consensus disturbance rejection scheme (13). The communication graphs are shown in Fig. 1 with unitary edge weights. In both example, the initial x_i, χ_i, ω_i, z_i are generated randomly via Gaussian distribution, and the initial c_{ij} are set as 0.1, $\forall i, j \in \mathcal{I}_N$.



(a) $\mathcal{G}_1$ for Example 1 (b) $\mathcal{G}_2$ for Example 2

Fig. 1: The communication graphs where the DSTs are highlighted with red color.

Example 1 *Consider a multiagent system consisting of 6 second-order agents where*

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \ B = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$
$$D = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \ E = \begin{pmatrix} 0 & 1.5 \\ -0.8 & 0 \end{pmatrix}. \tag{22}$$

It can be verified that Assumption 1-3 hold, and the matrix $F = (0, 1)$. Solving LMIs (6)-(7) gives

$$P = \begin{pmatrix} 1.8320 & -0.4000 \\ -0.4000 & 1.8320 \end{pmatrix},$$

$$Q = \begin{pmatrix} 1.8104 & 0.0277 & -0.8625 & -0.2063 \\ 0.0277 & 1.7253 & -0.4713 & -1.4358 \\ -0.8625 & -0.4713 & 2.1824 & 0.5405 \\ -0.2063 & -1.4358 & 0.5405 & 3.6667 \end{pmatrix},$$

resulting in $K = (0.1251, 0.5732)$ and

$$\Gamma_x = \begin{pmatrix} 0.6906 & 0.0951 \\ 0.0951 & 0.8973 \end{pmatrix}, \ \Gamma_\omega = \begin{pmatrix} 0.2850 & 0.1484 \\ 0.0341 & 0.3348 \end{pmatrix}.$$

Finally, let $\gamma = 0.01$ in (13d). After the simulation, the states of the agents are shown in Fig. 2, where the agents reach consensus asymptotically. The coupling gains c_{ij} are shown in Fig. 3, where the gains on the DST keep updating before converging to some constants. It is interesting to note that the average coupling gain in the steady state network is $mean(0.4254, 0.4981, 1.0156, 0.7311, 0.6882, 0.1) = 0.5764$, which is fairly smaller than the sufficient bound provided in Lemma 3 (which is 2 in this case).

To highlight the necessity of introducing adaptive couplings, we show that the non-adaptive case with $\gamma = 0$ in (13d) fails to guarantee consensus in Fig. 4. In this case, the scheme is equivalent to (5) with $c = 0.1$ (the initial c_{ij}), which is too weak for the agents to collaborate over a directed ring: it is known that a directed ring structure is harmful for consensus [22].

Example 2 *Consider the problem of consensus disturbance rejection for formation control of unmanned aerial vehicles (UAVs) studied in [15]. Each agent represents a YF-22 research UAV under a possible vibration in the control sur-*

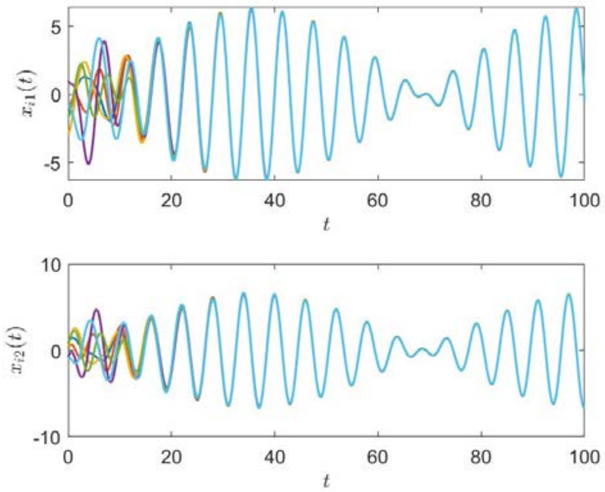


Fig. 2: The states x_i of the second-order agents under the proposed adaptive scheme (13) with $\gamma = 0.01$.

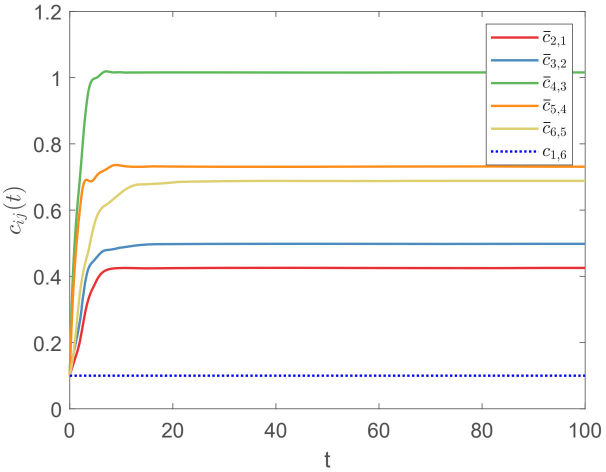


Fig. 3: The adaptive coupling gains c_{ij}.

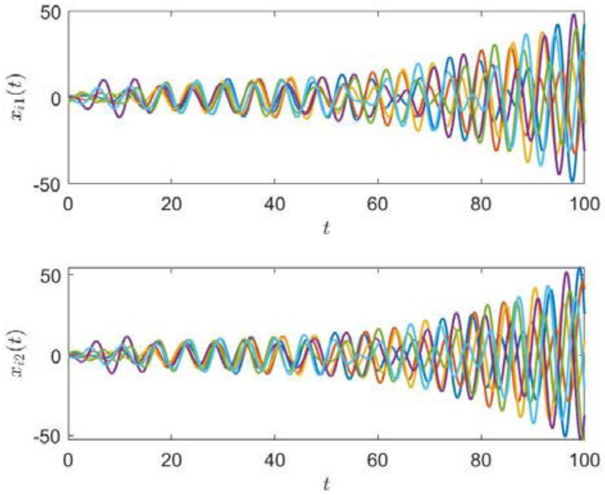


Fig. 4: The states x_i of the second-order agents under non-adaptive scheme $\big((13)$ with $\gamma = 0\big)$.

face, whose longitudinal dynamics can be identified by (1)-(2) with

$$A = \begin{pmatrix} -0.2840 & -23.0960 & 2.4200 & 9.9130 \\ 0 & -4.1170 & 0.8430 & 0.2720 \\ 0 & -33.8840 & -8.2630 & -19.5430 \\ 0 & 0 & 1.0000 & 0 \end{pmatrix},$$

$$B = \begin{pmatrix} 20.1680 \\ 0.5440 \\ -39.0850 \\ 0 \end{pmatrix}, \ E = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}.$$

Here, the four states of $x_i = (V_i, \alpha_i, q_i, \theta_i)$ *are the speed in meters per second, angle of attack in degrees, the pitch rate in degrees per second, and pitch in degrees, respectively. The vibrations, as disturbance* ω_i *to be rejected, has frequency 2 radians per second and satisfies the matching condition with* $F = (1, 0)$*. The communication graph is* $\mathcal{G}_2$ *shown in Fig. 1(b) with a DST.*

The gain matrices K *and* Γ *are designed following Theorem 1, and are omitted here for brevity. Let the parameter* $\gamma = 0.2$ *for adaptation. After implementing (13), the states of the UAVs reach consensus, as shown in Fig. 5. In this case, the flight formation is maintained under the disturbances, though the disturbances have an impact in the common trajectories. The adaptive coupling gains are shown in Fig. 6. Finally, the control inputs are shown in Fig. 7.*

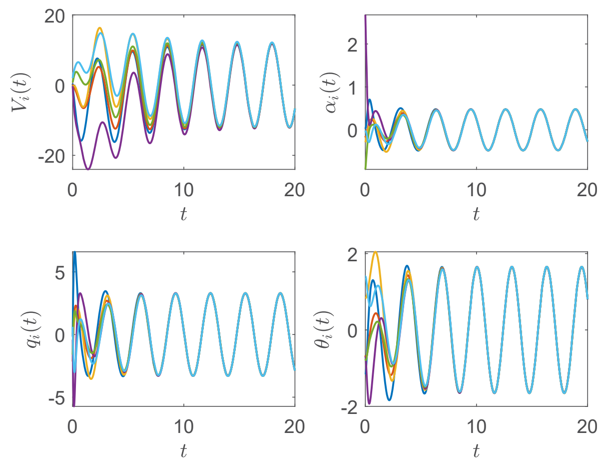


Fig. 5: The states x_i of the UAVs under the proposed adaptive scheme (13) with $\gamma = 0.2$.

5 Conclusions

In this paper, a novel directed spanning tree (DST) perspective has been presented to reconsider the consensus disturbance rejection for multiagent systems over digraphs. Specifically, we have reproduced the lower bound for a static homogeneous coupling gain in the literature by exploring the structure of a DST. Then, we have proposed a DST-based adaptive consensus disturbance rejection scheme, which eliminates the requirement for the global information of the Laplacian eigenvalues.

Future work may include nonlinear agents, and/or extend the results into a leader-follower network [20].

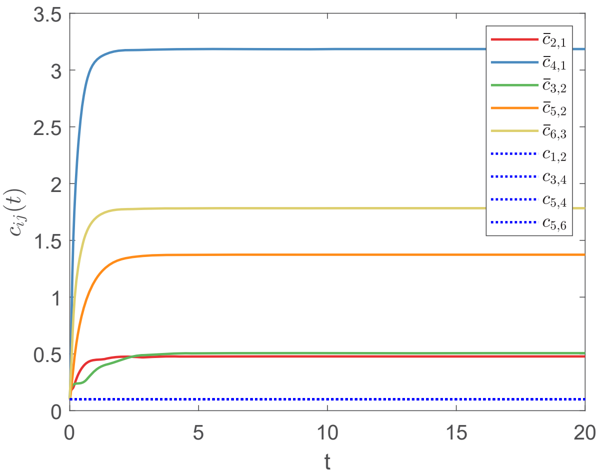


Fig. 6: The adaptive coupling gains c_{ij}.

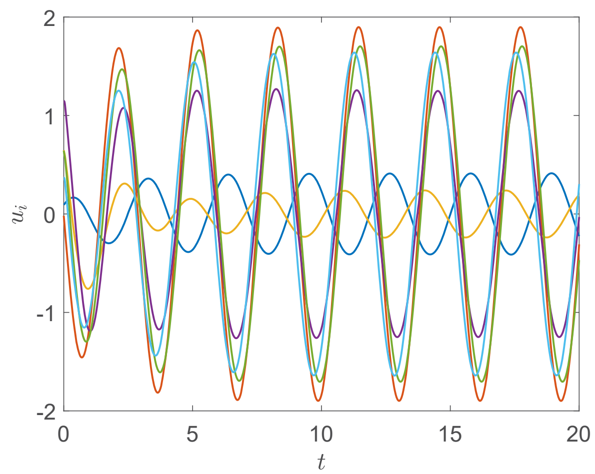


Fig. 7: The control inputs u_i for consensus disturbance rejection of the UAVs.

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