Robust Model Reference Adaptive Consensus with Neural Networks

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Abstract: This paper addresses distributed and robust leaderless consensus control for a class of uncertain multiagent systems with matched unknown nonlinearities and disturbances. The problem is challenging due to the lack of a leader (reference signal), the large uncertainties in agent dynamics, and the asymmetric communications among the agents. A novel neural network embedded model reference adaptive consensus (NN-MRACon) framework is proposed, which bridges NN and MRACon by means of nonsmooth control. Asymptotic consensus is proved based on robust analysis and input-to-state stability theory. Numerical examples on networks of second-order integrators and two-mass-spring systems are included to validate the effectiveness of NN-MRACon.

Key Words: Robust adaptive control, Consensus, Neural Networks, Nonsmooth control

1 Introduction

Consensus of multiagent systems appears in a variety of scenarios such as decision making [1, 2], optimization and economic dispatch [3, 4], industrial processes [5], etc. Depending on the absence or the presence of a reference agent, the formulation of consensus can be roughly divided into leaderless consensus [6–8] and leader-follower consensus [9, 10]: a critical point in the leaderless setting is that the consensus needs to be pursued by purely self-organizing behaviors of the agents themselves.

Uncertainties are inevitable in realistic systems. For a single system with unknown parameters or unknown systems matrices, many tools have been developed for regulation, tracking and parameter estimation, such as the celebrated model reference adaptive control [11]. When such a tool is applied to multiagent systems, several interesting results have been reported in recent years. For instance, it has been shown, for a network of linear heterogeneous harmonic oscillators with unknown frequencies, that leaderless consensus is attainable by adaptively learning a priori unknown group model [7]. For linear heterogeneous multiagent systems with unknown system matrices, it has been shown that adaptive leader-follower consensus is attainable by simultaneously adapting feedback gains and coupling gains [10]. More recently, a model reference adaptive consensus (MRACon) framework has been proposed in [12], which provides a solution for leaderless consensus of multiagent systems with matched unknown parameters, and is particularly effective when the communications graph among the agents is directed (asymmetric).

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For systems with more complex unmodelled dynamics and/or unknown disturbances, the methods in [7, 10, 12] may not be valid anymore. In this scenario, neural network (NN) adaptive control is recognized as a powerful tool, thanks to the ability of NN to approximate unknown nonlinearities. While classical NN based multiagent control methods only guarantee ultimately uniformly bounded coordination errors [13–15], state-of-the-art results have shown that it is possible to design asymptotic coordination control based on NN even though the uncertainties can not be completely neutralized. For instance, this was shown for consensus, tracking and containment control in [16, 17], and for formation control in [18]. Nevertheless, it should be noted that the communication graph in [16–18] is assumed directed balanced, or undirected connected. For general directed graph with a spanning tree, it is still an open problem (to our best knowledge) to design NN based asymptotic coordination control.

Motivated by the above discussions, we address the consensus control of a class of uncertain multiagent systems with unknown nonlinear dynamics and disturbances over a directed graph. The challenges are three fold, i.e., the lack of a leader (reference signal), the large uncertainties in agent dynamics, and the asymmetric communications among the agents. We propose a NN embedded MRACon framework, shorted as NN-MRACon, to tackle these challenges. The proposed NN-MRACon integrates both advantages of NN adaptive control of handling large uncertainties, and MRACon of addressing leaderless consensus with asymmetric communications.

2 Preliminaries

2.1 Notations and Graph Theory

A *directed graph* [19] (or simply *digraph*) $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ is specified by the node set $\mathcal{V} = \{1, 2, \cdots, N\}$, the edge set $\mathcal{E} = \{e_{ij}, i \neq j | i \to j\}$, and the adjacency matrix $\mathcal{A} =$

Notations:

$\mathbb{R}$ (resp. $\mathbb{R}^+$)	the set of real (resp. positive) scalars;
$\mathbb{R}^n$	the set of n-dimensional vectors;
$\mathbb{R}^{n\times m}$	the set of $n \times m$ matrices;
$\mathbf{I}_n$	$n \times n$ identity matrix;
$\mathbf{1}_n$	n-dimensional vector $(1, \cdots, 1)^T$;
0	zero scalar, zero vectors, and zero matrices;
tr(A)	the trace of matrix A;
$\lambda(A)$	the eigenvalue of matrix A;
$A > 0$	matrix A is positive definite;
null(A)	the zero space of matrix A;
span(a)	the space spanned by vector a;
sgn(a)	element-wise signum function of vector a;
$\|a\|$	the 2-norm of vector or matrix a;
$\|a\|_1$ (resp. $\|a\|_\infty$)	the 1-norm (resp. ∞-norm) of vector a;
$\otimes$	the Kronecker product;
$*$	the complex conjugate.

$(a_{ij}) \in \mathbb{R}^{N\times N}$ such that $a_{ij} > 0$ if $e_{ji} \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. The Laplacian matrix $\mathcal{L} = (l_{ij}) \in \mathbb{R}^{N\times N}$ associated with $\mathcal{G}$ is composed of $l_{ij} = -a_{ij}$ for $i \neq j$, and $l_{ii} = \sum_{j=1}^N a_{ij}$. For $e_{ij} \in \mathcal{E}$, i is called an in-neighbor of j. A path is a sequence of edges connecting a pair of nodes, which respects the edge directions.

A directed spanning tree $\bar{\mathcal{G}}(\mathcal{V}, \bar{\mathcal{E}}, \bar{\mathcal{A}})$ of $\mathcal{G}$ is a subgraph which contains a root (has no in-neighbors), such that one can find a unique path from the root to every other node. Without loss of generality, we label the root as node 1. Following the notations in [3, 20, 21], let i_k denote the unique in-neighbor of node $k+1$ in $\bar{\mathcal{G}}$, $k = 1, \cdots, N-1$.

2.2 Problem Setup

Consider a multiagent system with dynamics

$$\dot{x}_i(t) = Ax_i(t) + B(u_i(t) + f_i(x_i(t)) + d_i(t)), \quad (1)$$

where $x_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}^m$ are, respectively, the state and control input of agent i, $i = 1, \cdots, N$. The system matrices (A, B) are known, constant, stabilizable, and with compatible dimensions. The function $f_i(\cdot) : \mathbb{R}^n \to \mathbb{R}^m$ is unknown, and the disturbance $d_i(t) \in \mathbb{R}^m$ is unknown yet bounded, i.e.,

$$\|d_i(t)\|_\infty \leq \bar{d}_i. \quad (2)$$

The lumped uncertainties (f_i and d_i) are referred to as matched uncertainties since they appear in the same channel as u_i. The model can describe a variety of uncertain systems with linearized dynamics and matched uncertainties [15–18].

Since f_i is unknown, we propose to estimate the function using a NN. The following assumption is fairly standard in related literature [13, 14, 16].

Assumption 1 *The unknown function $f_i(x_i)$ for agent i in (1) can be linearly parameterized, over a sufficiently large compact set Ω_i, by an NN as*

$$f_i(x_i) = W_i^T S_i(x_i) + \epsilon_i, \quad (3)$$

where $W_i \in \mathbb{R}^{q\times m}$ is an ideal weight matrix which is bounded, and $S_i(x_i) : \mathbb{R}^n \mapsto \mathbb{R}^q$ is the stacked vector of activation functions of the q hidden layer neurons, which is also bounded.

Under Assumption 1, the Stone-Weierstrass theorem [22] guarantees that the approximation error ϵ_i can be arbitrarily small. Specifically, for any $\forall x_i \in \Omega_i$, there holds

$$\|\epsilon_i(x_i)\|_\infty \leq \bar{\epsilon}_i, \quad (4)$$

for any pre-specified $\bar{\epsilon}_i > 0$, provided q being sufficiently large.

The communication topology among the agents is characterized by a digraph $\mathcal{G}$ which satisfies the following mild assumption in consensus control [19].

Assumption 2 *The communication digraph $\mathcal{G}$ contains a directed spanning tree $\bar{\mathcal{G}}$.*

Under Assumption 2, let us construct two matrices based on the directed spanning tree $\bar{\mathcal{G}}$. Define $\Xi \in \mathbb{R}^{(N-1)\times N}$ as

$$\Xi_{kj} = \begin{cases} -1, & \text{if } j = k+1, \\ 1, & \text{if } j = i_k, \\ 0, & \text{otherwise.} \end{cases} \quad (5)$$

In fact, Ξ is the difference matrix along the tree: $\Xi x = (x_{i_1} - x_2, x_{i_2} - x_3, \cdots, x_{i_{N-1}} - x_N)^T$. Define $Q \in \mathbb{R}^{(N-1)\times(N-1)}$ as

$$Q_{kj} = \sum_{c\in\bar{\mathcal{V}}_{j+1}} (\mathcal{L}_{k+1,c} - \mathcal{L}_{i_k,c}), \quad (6)$$

where $\bar{\mathcal{V}}_{j+1}$ represents the vertex set of the subtree of $\bar{\mathcal{G}}$ rooting at node $j+1$.

Proposition 1 ([3, 21]) *Under Assumption 2, the following statements hold for $\mathcal{L}$ (of $\mathcal{G}$), and Ξ, Q defined above:*

1. *$0 = \lambda_1(\mathcal{L}) < \Re(\lambda_2(\mathcal{L})) \leq \Re(\lambda_3(\mathcal{L})) \leq \cdots \leq \Re(\lambda_N(\mathcal{L}))$. Moreover, null$(\mathcal{L})$ = span$(\mathbf{1}_N)$.*
2. *$\Xi\mathcal{L} = Q\Xi$. Moreover, null(Ξ) = span$(\mathbf{1}_N)$.*
3. *$\lambda_i(Q) = \lambda_{i+1}(\mathcal{L})$, $i = 1, \cdots, N-1$.*

Our goal is that, despite the uncertainties of the agents and the asymmetry of the communication digraph, the agents reach consensus asymptotically, i.e., $\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0, \forall i, j \in \mathcal{V}$.

3 NN-MRACon: Controller and Convergence

3.1 Controller Design

The framework of NN-MRACon is illustrated as follows. We endow each agent a reference model with transfer function $\frac{B}{s\mathbf{I}-A}$, i.e., the intrinsic linear dynamics without uncertainties. This decomposes the consensus problem into two parts: the decentralized tracking of the agents to the corresponding reference models by NN adaptive control, and the consensus over the linear reference models. The latter is attainable by referring to input-to-state stability theory.

Denote $\delta_i = \sum_{j=1}^N a_{ij}(x_i - x_j)$ as the accumulative relative error with respect to its in-neighbors, and $e_i = x_i - z_i$

as the decentralized local tracking error. The control law is proposed for agent $i \in \mathcal{V}$ as:

$$u_i = Ke_i + \alpha_i \text{sgn}(Ke_i) + \beta K \delta_i - \widehat{W}_i^T S_i(x_i) \quad (7a)$$
$$\dot{z}_i = Az_i + \beta BK\delta_i \quad (7b)$$
$$\dot{\widehat{W}}_i = \gamma_i (S_i(x_i) e_i^T P^{-1} B - \sigma_i(\widehat{W}_i - \overline{W}_i)) \quad (7c)$$
$$\dot{\overline{W}}_i = \sigma_i \tau_i (\widehat{W}_i - \overline{W}_i), \quad (7d)$$

where K is a gain matrix, and $\alpha_i, \beta, \gamma_i, \sigma_i, \tau_i \in \mathbb{R}^+$. Here, $\widehat{W}_i$ is the estimation of the unknown weight matrix W_i and $\overline{W}_i$ is the pseudo ideal weight matrix for the NN approximator of agent i [16]. The overall control diagram for agent $i \in \mathcal{V}$ is shown in Fig. 1.

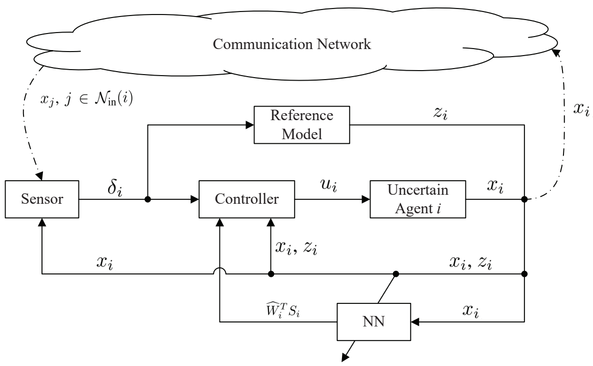

Figure 1: NN-MRACon: the control block diagram.

3.2 Convergence Result

We are now in the position to state the main convergence result of NN-MRACon.

Theorem 1 *Under Assumptions 1 and 2, the asymptotic consensus problem of MAS (1) can be solved under NN-MRACon controller (7) with $K = -B^T P^{-1}$, $\alpha_i \geq \bar{d}_i + \bar{\epsilon}_i$, $\beta \geq \frac{1}{\Re(\lambda_2(\mathcal{L}))}$, and $\gamma_i, \sigma_i, \tau_i \in \mathbb{R}^+$. Here $P > 0$ is a solution to the following LMI:*

$$AP + PA^T - 2BB^T < 0. \quad (8)$$

Proof. In consistently with the illustrations of the NN-MRAcon framework in previous subsection, the proof will be divided into two parts, i.e., the convergence of e_i and the consensus of z_i.

a) Convergence of e_i. By substituting (3), (7a) to (1), the dynamics of e_i is

$$\begin{aligned}\dot{e}_i =&(A + BK)e_i + \alpha_i B \text{sgn}(Ke_i) \\ &- B(\tilde{W}_i^T S_i(x_i) - d_i - \epsilon_i)\end{aligned} \quad (9)$$

where $\tilde{W}_i = \widehat{W}_i - W_i$. Consider, for each $i \in \mathcal{V}$, the candidate Lyapunov function

$$V_i = e_i^T P^{-1} e_i + \text{tr}(\frac{1}{\gamma_i} \tilde{W}_i^T \tilde{W}_i) + \text{tr}(\frac{1}{\tau_i} \tilde{\overline{W}}_i^T \tilde{\overline{W}}_i) \quad (10)$$

where P is a solution of (8), and $\tilde{\overline{W}}_i = \overline{W}_i - W_i$. Combined with (7c)-(7d), the derivative of V_i along the trajectory of (9) is

$$\begin{aligned}\dot{V}_i =&2e_i^T P^{-1}(A + BK)e_i + 2\alpha_i e_i^T P^{-1} B \text{sgn}(Ke_i) \\ &- 2e_i^T P^{-1} B \tilde{W}_i^T S_i(x_i) + 2e_i^T P^{-1} B(d_i + \epsilon_i) \\ &+ 2\text{tr}\big(\tilde{W}_i^T (S_i(x_i) e_i^T P^{-1} B - \sigma_i(\widehat{W}_i - \overline{W}_i))\big) \\ &+ 2\text{tr}(\sigma_i \tilde{\overline{W}}_i^T (\widehat{W}_i - \overline{W}_i)).\end{aligned} \quad (11)$$

Since $\text{tr}(XY) = \text{tr}(YX)$ for any compatible matrices X and Y, an immediate observation is that

$$\text{tr}\big(\tilde{W}_i^T S_i(x_i) e_i^T P^{-1} B\big) = e_i^T P^{-1} B \tilde{W}_i^T S_i(x_i) \quad (12)$$

Note that $K = -B^T P^{-1}$, and $a^T \text{sgn}(a) = \|a\|_1$ for any vector a. Then, we have

$$\alpha_i e_i^T P^{-1} B \text{sgn}(Ke_i) = -\alpha_i \|Ke_i\|_1. \quad (13)$$

Moreover, the Hölder inequality guarantees that

$$\begin{aligned}e_i^T P^{-1} B(d_i + \epsilon_i) &\leq \|Ke_i\|_1 \|d_i + \epsilon_i\|_\infty \\ &\leq (\bar{d}_i + \bar{\epsilon}_i)\|Ke_i\|_1\end{aligned} \quad (14)$$

where we have used (2) and (4) to obtain the last inequality. Then, it follows from (11)-(14) that

$$\begin{aligned}\dot{V}_i \leq&2e_i^T P^{-1}(A + BK)e_i - 2(\alpha_i - \bar{d}_i - \bar{\epsilon}_i)\|Ke_i\|_1 \\ &- 2\sigma_i \text{tr}\big((\widehat{W}_i - \overline{W}_i)^T ((\widehat{W}_i - \overline{W}_i))\big) \\ \leq&2e_i^T P^{-1}(A + BK)e_i.\end{aligned} \quad (15)$$

Let $\bar{e}_i = P^{-1} e_i$. Then, the above is equal to

$$\begin{aligned}\dot{V}_i \leq&2\bar{e}_i^T (A + BK)P\bar{e}_i \\ =&\bar{e}_i^T (AP + PA^T - 2BB^T)\bar{e}_i \leq 0.\end{aligned} \quad (16)$$

Since $\dot{V}_i \leq 0$, $V_i(t)$ is bounded, implying that e_i, $\tilde{W}_i$, $\tilde{\overline{W}}_i$ are bounded. Note that $\dot{V}_i \equiv 0$ gives $\bar{e}_i = 0$, thus $e_i = 0$. By LaSalle's invariance principle [23], it follows that $\lim_{t\to\infty} e_i(t) = 0$, $i \in \mathcal{V}$. That is, the decentralized tracking is achieved.

b) Consensus of z_i. Let us stack the variables as $z = (z_1^T, \cdots, z_N^T)^T$, and δ, x, e is a similar way. Then, it is clear that $\delta = (\mathcal{L} \otimes \mathbf{I}_n)x$. By (7b), we have

$$\begin{aligned}\dot{z} &= (\mathbf{I}_N \otimes A)z + \beta(\mathcal{L} \otimes BK)x \\ &= (\mathbf{I}_N \otimes A + \beta\mathcal{L} \otimes BK)z + \beta(\mathcal{L} \otimes BK)e\end{aligned} \quad (17)$$

Let $\bar{z} = (\Xi \otimes \mathbf{I}_n)z$ where Ξ is defined as in (5). Based on statement 2 of Proposition 1, we have

$$\dot{\bar{z}} = (\mathbf{I}_N \otimes A + \beta Q \otimes BK)\bar{z} + \beta(\Xi\mathcal{L} \otimes BK)e \quad (18)$$

where Q is defined as in (6). Moreover, $\bar{z} = 0$ if and only if z_i reach consensus. Therefore, it is sufficient to prove that the system (18) is input-to-state stable with respect to the decaying input e, which requires to show that the matrix $\mathbf{I}_N \otimes A + \beta Q \otimes BK$ is Hurwitz. Equivalently, it suffices to show that the subsystems $A + \beta\lambda_i(\mathcal{L})BK$ for

$i \in \{2, \cdots, N\}$ are Hurwitz (recall statement 3 of Proposition 1). Note that

$$\begin{aligned}&(A+\beta\lambda_i(\mathcal{L})BK)P+P(A+\beta\lambda_i(\mathcal{L})BK)^*\\=&AP-\beta\lambda_i(\mathcal{L})BB^T+PA^T-\beta\lambda_i^*(\mathcal{L})BB^T\\=&AP+PA^T-2\beta\Re(\lambda_i(\mathcal{L}))BB^T\\\leq&AP+PA^T-2BB^T<0.\end{aligned} \tag{19}$$

The above implies that $A+\beta\lambda_i(\mathcal{L})BK$ for $i \in \{2, \cdots, N\}$ are indeed Hurwitz. Then, we can conclude that z_i reach consensus which, together with the convergence of e_i, guarantees that $\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0, \forall i, j \in \mathcal{V}$. This concludes the proof. ■

Remark 1 (On the pseudo ideal weight $\overline{W}_i$) *As e_i decays to zero, the NN weight $\widehat{W}_i$ converges to the corresponding pseudo ideal weight $\overline{W}_i$ [16–18]. In fact, it follows from (7c)-(7d) that $\dot{\hat{W}}_i^e = -\sigma_i(\gamma_i + \tau_i)\hat{W}_i^e + \gamma_i S_i(x_i)e_i^T P^{-1}B$ where $\hat{W}_i^e = \hat{W}_i - \overline{W}_i$. Since $S_i(x_i)$ is bounded and σ_i, γ_i, $\tau_i \in \mathbb{R}^+$, there holds $\lim_{t\to\infty}(\hat{W}_i - \overline{W}_i) = 0$ for all i. It should be clarified that the convergence of $\widehat{W}_i$ to $\overline{W}_i$ does not necessarily imply the convergence to the ideal weight W_i, nor zero approximation error to the unknown f_i. It is the introduction of $\overline{W}_i$, incorporated with nonsmooth control, that allows to conclude the asymptotic stability of e_i, which further paves the way to conclude the asymptotic consensus of z_i by input-to-state stability theory.*

Remark 2 (On the matrix Q) *In the recent work of MRACon [12], a matrix $\tilde{Q} \in \mathbb{R}^{(N-1)\times N}$ is defined as*

$$\tilde{Q} = \begin{pmatrix} -1+(N-1)v & 1-v & -v & \dots & -v \\ -1+(N-1)v & -v & 1-v & \ddots & \vdots \\ -1+(N-1)v & \vdots & \ddots & \ddots & -v \\ -1+(N-1)v & -v & \dots & -v & 1-v \end{pmatrix}$$

with $v = \frac{N-\sqrt{N}}{N(N-1)}$. The matrix $\tilde{Q}$ satisfies $\lambda_i(\tilde{Q}\mathcal{L}\tilde{Q}) = \lambda_{i+1}(\mathcal{L})$, $i = 1, \cdots, N-1$. Clearly, $\tilde{Q}$ and Q in (6) play the same role, i.e., to seek a sufficient lower bound of β in term of the algebraic connectivity $\lambda_2(\mathcal{L})$. The difference is that, in our case, Q is constructed by fully exploring Assumption 2. The reduced-order square Q fully inherits the information of $\mathcal{L}$ through a commutative-like law ($\Xi\mathcal{L} = Q\Xi$), and is commonly used in directed spanning tree based adaptive control literature [20, 21].

Remark 3 (On the nonsmoothness of u_i) *The dynamics of e_i is discontinuous due to the presence of the signum function. Since the signum function is measurable and essentially bounded, the solutions for (9) always exist in the sense of Filippov [24]. Note further that the candidate Lyapunov function (10) is continuous differentiable and its set-valued Lie derivative is a singleton at the discontinuous point, which means that stability analysis can be carried out without introducing differential inclusions [18, 25].*
In practice, the chattering phenomena can be reduced by replacing sgn(x) with the nonlinear function $\varpi(x,t) = \frac{x}{\|x\|+\eta exp(-\rho t)}$, or simply $\varpi(x) = \frac{x}{\|x\|+\rho}$ where $\eta, \rho \in \mathbb{R}^+$. In fact, the above are continuous time-varying and time-invariant approximations, respectively, of the signum function based on the boundary layer concept [26, 27].

4 Examples

Example 1 *Consider a network of 6 second-order integrator agents with*

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},\ B = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$
$$f_i(x_i(t)) = ix_{i1}^2,\ d_i(t) = 0.2\sin(it),$$

where $i = 1, \cdots, 6$. The communication graph is a directed ring with edge set $\mathcal{E} = \{e_{12}, e_{23}, e_{34}, e_{45}, e_{56}, e_{61}\}$: in this case, $\Re(\lambda_2(\mathcal{L})) = 0.5$. The upper bound of disturbance is $\bar{d}_i = 0.2$.
In the control design phase, the above f_i and d_i are assumed unknown. In order to estimate f_i, we endow each agent i a NN with randomly initialized input layer weight W_i^{input}, and 6 hidden neurons activated by sigmoid function. That is, the estimated function $\hat{f}_i = \widehat{W}_i S_i(x_i)$ where $[S_i(x_i)]_j = \frac{1}{1+exp(-[W_i^{input}x_i]_j)}$. Here $[\cdot]_j$ is the j-th component associated with the j-th neuron. The approximation error is specified as $\bar{\epsilon}_i = 0.1$. Solving LMI (8) gives

$$P = \begin{pmatrix} 1.7559 & -0.5853 \\ -0.5853 & 0.5853 \end{pmatrix},$$

which leads to the feedback gain matrix $K = (-0.8543, -2.5628)$. Set the parameters as $\alpha_i = 0.3$, $\beta = 3$, $\gamma_i = \tau_i = 200$, $\sigma_i = 0.6$.
The initial x_i and z_i follow Gaussian distribution with standard deviation 3. The initial $\widehat{W}_i$ and $\overline{W}_i$ are chosen as zero matrices. After implementing the NN-MRACon controller (7), the agents reach asymptotical consensus in both positions and velocities, as shown in Fig. 2. The actual functions f_i and the estimated $\hat{f}_i$ are shown in Fig. 3, where the unknown functions f_i are quite well approximated by the NN. Finally, the control inputs are shown in Fig. 4.

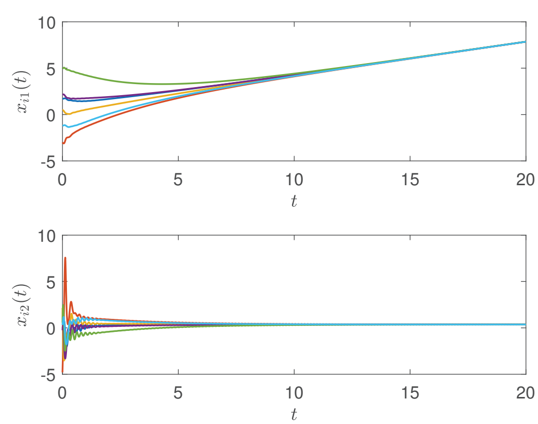

Figure 2: The states x_i of the agents.

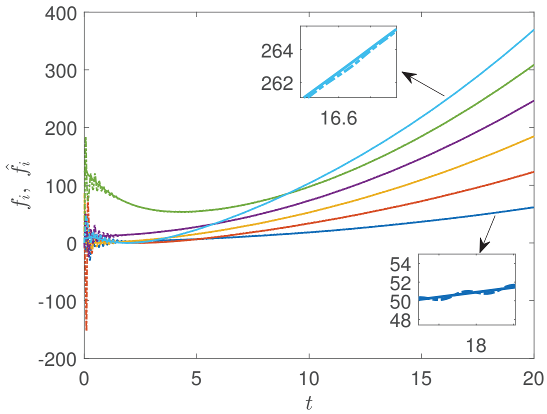

Figure 3: The actual f_i (solid line) and its estimation $\hat{f}_i$ (dashed line) of the agents.

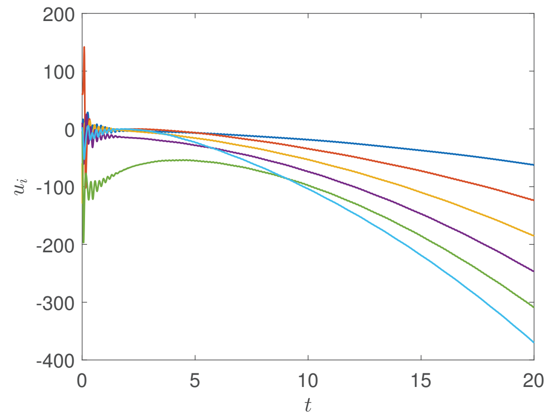

Figure 4: The control inputs u_i of the agents.

Example 2 *Consider a network of* 6 *uncertain two-mass-spring systems [28], see also Fig. 5. The dynamics of each system can be modelled by (1) with*

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_1-k_2}{m_1} & 0 & \frac{k_2}{m_2} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & 0 & \frac{-k_2}{m_2} & 0 \end{pmatrix},\ B = \begin{pmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{pmatrix},$$
$$f_i(x_i(t)) = x_{i2}^2 \cos(x_{i1}),\ d_i(t) \equiv 0.1,$$

where $m_1 = 1.1$, $m_2 = 0.9$ *are two masses,* $k_1 = 1.0$, $k_2 = 1.5$ *are spring constants. The components of the states represent the displacement and velocity of the first mass, followed by those of the second mass, respectively. A single torque is applied to the first mass, along with unknown matched dynamics* f_i *and disturbance* d_i *(which is a constant bias in this case). A solution to LMI (8) is*

$$P = \begin{pmatrix} 0.5793 & -0.1875 & 0.3338 & -0.2647 \\ -0.1875 & 0.7966 & 0.0307 & -0.2728 \\ 0.3338 & 0.0307 & 0.7007 & -0.1136 \\ -0.2647 & -0.2728 & -0.1136 & 0.6974 \end{pmatrix},$$

resulting in $K = (-1.5856, -1.9740, 0.6357, -1.2705)$. *The structure of the communication network as well as the NN are assumed the same as those in the above example. The control parameters are selected as* $\alpha_i = 0.2$, $\beta = 3$, $\gamma_i = \tau_i = 100$, $\sigma_i = 0.5$.

After implementing the NN-MRACon controller (7), the uncertain two-mass-spring systems reach asymptotical consensus, as shown in Fig. 6. The estimation performance and control inputs are omitted for brevity. Instead, we include a Fig. 7 to show the convergence of $\widehat{W}_i$ *to the corresponding pseudo ideal weight* $\overline{W}_i$*, which validates the discussions in Remark 1.*

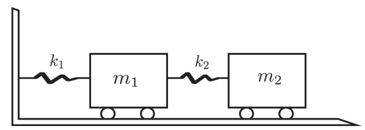

Figure 5: Two-mass-spring system.

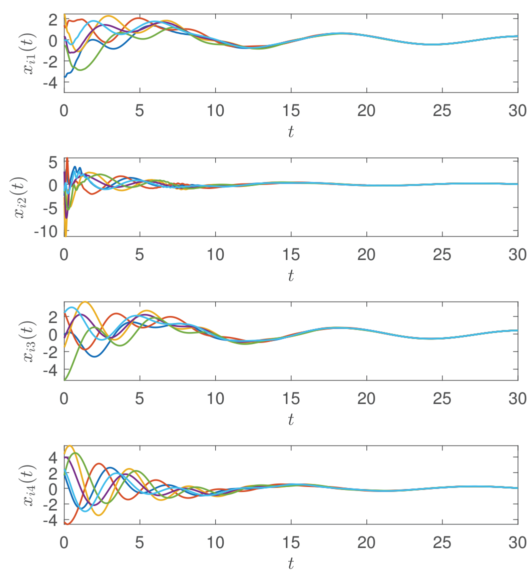

Figure 6: The states x_i of the agents.

5 Conclusions and future works

A neural network embedded model reference adaptive consensus (NN-MRACon) framework has been proposed for leaderless consensus of multiagent systems, where the only assumption on the communication graph is the existence of a directed spanning tree. Upon selecting appropriate control gains, the asymptotic consensus of the agents has been proved, in spite of the presence of matched unknown nonlinearities and unknown disturbances.

Fully distributed NN based control without the global knowledge of the Laplacian spectrum was studied in [17, 18] for consensus and formation, respectively. However,

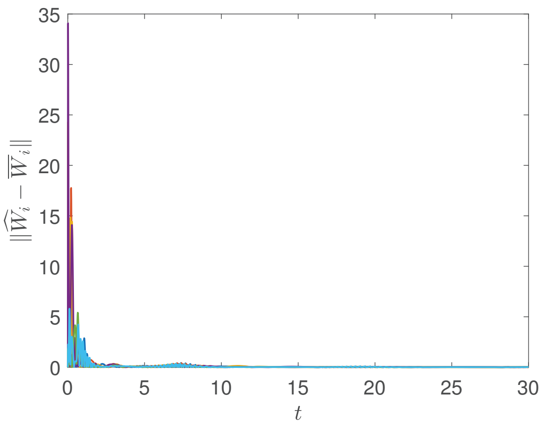

Figure 7: The pseudo convergence errors $\|\widehat{W}_i - \overline{W}_i\|$ of the agents.

the results are limited in undirected graphs. For general digraphs, the extension of NN-MRACon to a fully distributed version seems not trivial, and awaits future research.

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