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D-MRACO: Decentralized Model Reference Adaptive Controller and Observer

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Abstract: Model Reference Adaptive Controller and Observer (MRACO) has been proposed recently in Ohrem and Holden (2021) for a single unknown plant. Motivated by the advances in multiagent systems, we propose a unified framework named Decentralized MRACO (D-MRACO), addressing leaderless synchronization and leader-follower tracking problems of multiple unknown plants (agents). Numerical examples show the efficiency and robustness of the proposed D-MRACO.

Keywords: Multiagent systems, MRAC, Distributed observer, Output feedback

1. INTRODUCTION

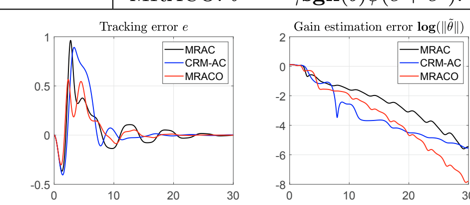
Abbreviations:	
MRAC	Model Reference Adaptive Control
ORM	Open-loop Reference Model
CRM	Closed-loop Reference Model
CRM-AC	CRM based Adaptive Control
MRACO	Model Reference Adaptive Controller and Observer
MRACon	Model Reference Adaptive Consensus
D-MRACO	**Decentralized MRACO**

MRAC is a longstanding and powerful tool for controlling uncertain systems, with a goal of tracking a reference model specified by the designer: this reference model is traditionally an open-loop reference model Ioannou and Sun (2012).

With the CRM-AC in Gibson et al. (2013, 2015), it was shown that tracking a ORM can be realized based on a CRM, where the CRM plays a role of an observer, thus allowing output feedback. Then, the tracking problem can be separated into two parts, i.e., the tracking to the CRM, and the convergence of the CRM to the desired ORM. One can think about CRM-AC as a generalization of MRAC by introducing an additional degree of freedom represented by the observer gain. Indeed, CRM-AC provides better transient behavior as compared with MRAC, nevertheless it may tend to peaking unless the observer gain is chosen with care Gibson et al. (2013). As an extension of CRM-AC, the authors of Ohrem and Holden (2021) proposed a MRACO framework based on both open and closed-loop reference models, where the key difference is to decouple the tracking to the CRM and the tracking to the ORM. As such, the tracking to the desired ORM will be less sensitive to the observer errors by CRM. To clarify, we sketch MRAC, CRM-AC, and MRACO for uncertain scalar plants with state feedback in Table 1. One can clearly see that MRACO encompasses classic MRAC and CRM-AC, and exhibits smoother transients in target tracking and faster convergence in parameter estimation.

Table 1. MRAC, CRM-AC, and MRACO for uncertain scalar plants with state feedback.

Plant	$\dot{x} = ax + bu$; only $\mathbf{sgn}(b)$ is known, $b \neq 0$ (controllable).
ORM (Target)	$\dot{x}_m = a_m x + b_m r$; $a_m < 0, b_m, r$ are specified, $r(t)$ is bounded.
CRM	$\dot{x}_m^c = a_m x + b_m r - \rho(x - x_m^c)$; $\rho < 0$.
Controller	$u = \phi^T \theta(t) \triangleq k(t)x + l(t)r$; $\phi = (x, r)^T$, $\theta = (k, l)^T$,
Adaptive Law	MRAC: $\dot{\theta} = -\gamma \mathbf{sgn}(b)\phi e$, $e = x - x_m$; CRM-AC: $\dot{\theta} = -\gamma \mathbf{sgn}(b)\phi e^c$, $e^c = x - x_m^c$; MRACO: $\dot{\theta} = -\gamma \mathbf{sgn}(b)\phi(e + e^c)$.



Simulation results with $a = 1$, $b = 2$; $a_m = -1$, $b_m = 1$, $r = \sin(t)$; $\rho = -1$; $\gamma = 1$. Here, $\tilde{\theta}(t) = \theta(t) - \theta^*$ where $\theta^* \triangleq (\frac{a_m - a}{b}, \frac{b_m}{b})^T$ are ideal gains.

The above results are for a single uncertain plant. When MRAC is applied to multiagent systems, several interesting results have been reported in recent years. For instance, it has been shown, for a network of linear heterogeneous harmonic oscillators with unknown frequencies, that leaderless synchronization is attainable by adaptively learning a priori unknown group model Baldi and Frasca (2019). For linear heterogeneous agents with unknown dynamics, it has been shown that leader-follower tracking is attainable via hierarchical distributed model reference adaptation Baldi et al. (2018), or via adaptation in both feedback gains and coupling gains Azzollini et al. (2020). More recently, a MRACon framework has been proposed in Mei et al. (2021), which provides a solution for leaderless

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synchronization of linear agents with matched unknown parameters, and is particularly effective when the communications graph among the agents is directed (asymmetric). The idea of CRM-AC has also been adapted into a network setting to address leader-follower tracking in Goel and Roy (2021); Goel et al. (2022). Clearly, a MRACO design and analysis for multiagent systems is missing, with or without a leader, which motivates this work.

We propose a framework named D-MRACO that can handle both leaderless synchronization and leader-follower tracking problems in a unified way. D-MRACO separates the cooperative tasks into two subproblems, i.e., decentralized tracking control of the unknown agents to the according ORMs via MRACO, and observer-based distributed leaderless synchronization or leader-follower tracking control of the ORMs. As a consequence, D-MRACO allows distributed output feedback of the unknown agents, i.e., only the output information of the agent itself and its neighbors are needed. Note that this feature is absent in the MRACon framework proposed in Mei et al. (2021). Besides, the communication topology among the agents is supposed to be a general directed graph that contains a directed spanning tree, while most state-of-the-art MRAC methods in multiagent systems only apply to the undirected case (see for instance the aforementioned Baldi and Frasca (2019); Baldi et al. (2018); Azzollini et al. (2020); Goel and Roy (2021); Goel et al. (2022)).

Let us close this section by reviewing some notations of graph theory. A directed graph (or simply *digraph*) is denoted as $\mathcal{G}(\mathcal{V},\mathcal{E},\mathcal{A})$ where $\mathcal{V}$ is the node set, $\mathcal{E}=\{e_{ij}|i\to j, i\neq j\}$ is the edge set, and $\mathcal{A}=(a_{ij})\in\mathbb{R}^{N\times N}$ is the adjacency matrix such that $a_{ij}=1$ if $e_{ji}\in\mathcal{E}$, and $a_{ij}=0$ otherwise. The Laplacian matrix $\mathcal{L}=(l_{ij})\in\mathbb{R}^{N\times N}$ associated with $\mathcal{G}$ consists of $l_{ij}=-a_{ij}$ for $i\neq j$, and $l_{ii}=\sum_{j=1}^{N}a_{ij}$. For $e_{ij}\in\mathcal{E}$, node j can have access to the information of node i, meaning that i is a neighbor of j. A path is a sequence of edges connecting a pair of nodes, which respects the edge directions. A directed spanning tree of $\mathcal{G}$ is a subgraph with same nodes and selective edges from $\mathcal{G}$, such that there exists a root (has no neighbors) and one can find a unique path from the root to every other node. The matrix $\mathcal{L}$ has a simple zero eigenvalue and the rest eigenvalues have positive real parts, if and only if $\mathcal{G}$ contains a directed spanning tree (Ren and Beard, 2008, Lemma 2.4). In this case, we use $\lambda_2(\mathcal{L})$ to denote the eigenvalue of $\mathcal{L}$ with the minimum nonzero real part.

Notations: Denote $\mathbb{R}$ as the real space (of scalars), with superscript indicating dimensions of column vectors and matrices. For a complex scalar λ, denote $\Re(\lambda)$ as its real part. Let the superscript $\bullet^T$ be the real transpose and $\bullet^*$ be the complex conjugate transpose. Let I be the identity matrix, with subscript indicating dimensions. Let $\lambda_1(A)$ denote the eigenvalue of a square matrix A with the minimum real part.

2. PROBLEM FORMULATION

Consider a network of linear time invariant agents where the dynamics of the i-th ($i\in\{1,2,\cdots,N\}$) agent follows

$$\begin{aligned}\dot{x}_i &= Ax_i + B\Lambda u_i\\ y_i &= C^T x_i.\end{aligned}\tag{1}$$

in which $x_i\in\mathbb{R}^n$ is the state, and $u_i, y_i\in\mathbb{R}^m$ are the control input and the measured output with $m\leq n$. The matrices $A\in\mathbb{R}^{n\times n}$ and $\Lambda\in\mathbb{R}^{m\times m}$ are assumed to be *unknown*, and the matrices $B, C\in\mathbb{R}^{n\times m}$ are known.

Assumption 1. (Matching condition). There exists a $K^*\in\mathbb{R}^{n\times m}$ such that $A+B\Lambda K^{*T}=A_{\mathrm{m}}$ for a known matrix A_{m}, and $\Lambda K^{*T}\in\mathcal{D}$ for a known uncertainty set $\mathcal{D}$.

Assumption 2. (KYP condition). The triple (A_{m}, B, C^T) is controllable and observable, and the transfer function $G_{\mathrm{m}}(s)=C^T(sI-A_{\mathrm{m}})^{-1}B$ is strictly positive real (SPR).

Assumption 3. (Input uncertainty). Λ is diagonal with positive elements.

Remark 1. In Assumption 1, the matrix A_{m} represents known desirable dynamics, and $\mathcal{D}$ in practice means the confidence interval by the designer. Assumption 2 is standard in output feedback adaptive control Ioannou and Sun (2012); Narendra and Annaswamy (2012) and furthermore assures a solution in terms of a class of LMI condition. Assumption 3 can be extended to the case that Λ is either positive definite or negative definite with $\mathbf{sgn}(\Lambda)$ *priori* known. Assumptions 1-3 are standard in the literature Gibson et al. (2015); Ohrem and Holden (2021).

In this paper, we will address two scenarios of network setting, i.e., with and without a leader (reference signal):

Scenario I: Leaderless Synchronization. The interaction graph between the agents (1) is denoted by $\mathcal{G}(\mathcal{V},\mathcal{E},\mathcal{A})$ where $\mathcal{V}=\{1,2,\cdots,N\}$. The communication digraph $\mathcal{G}$ contains a directed spanning tree. The control objective is to design an output-feedback law u_i in (1) such that $x_i(t)\to x_j(t)$ for any i,j as $t\to\infty$.

Scenario II: Leader-follower Tracking. The agents in (1) are also referred to as the followers, and their communication topology is denoted by $\mathcal{G}$ as above. There is a leader agent modelled by

$$\begin{aligned}\dot{x}_0 &= A_{\mathrm{m}}x_0 + Br_0\\ y_0 &= C^T x_0.\end{aligned}\tag{2}$$

with $x_0\in\mathbb{R}^n$ being the leader's state, $r_0\in\mathbb{R}^m$ being its external input which is bounded, and $y_0\in\mathbb{R}^m$ being its output. The overall interaction graph between the followers (1) and the leader (2) is denoted by $\bar{\mathcal{G}}(\bar{\mathcal{V}},\bar{\mathcal{E}},\bar{\mathcal{A}})$ where $\bar{\mathcal{V}}=\{0,1,2,\cdots,N\}$. The communication digraph $\bar{\mathcal{G}}$ contains a directed spanning tree with the leader being the root. The control objective is to design an output-feedback law u_i such that $x_i(t)\to x_0(t)$ for any i as $t\to\infty$.

Note that, although the leaderless synchronization and leader-follower tracking are specified with respect to the state, the feedback to be designed is based on output information.

3. PRELIMINARY ANALYSIS

For each agent $i\in\mathcal{V}$, design a ORM as

$$\begin{aligned}\dot{x}_i^{\mathrm{m}} &= A_{\mathrm{m}}x_i^{\mathrm{m}} + Br_i\\ y_i^{\mathrm{m}} &= C^T x_i^{\mathrm{m}}\end{aligned}\tag{3}$$

where r_i is the input to be designed later. Then, the according CRM is designed as

$$\begin{aligned}\dot{\hat{x}}_i &= A_{\mathrm{m}}\hat{x}_i + Br_i - F(y_i-\hat{y}_i)\\ \hat{y}_i &= C^T\hat{x}_i\end{aligned}\tag{4}$$

with observer gain matrix F to be designed.

Now, we shall briefly illustrate the idea behind our solution. We endow each agent a ORM with inputs to be designed, and a corresponding CRM to allow output feedback. This decomposes Scenario I and Scenario II into two stages: first, promote consensus over the ORMs in Scenario I ($x_i^{\mathrm{m}} \to x_j^{\mathrm{m}}$) or promote tracking to the leader in Scenario II ($x_i^{\mathrm{m}} \to x_0$) by manipulating r_i, and, second, let each agent track the corresponding ORM by MRACO ($x_i \to x_i^{\mathrm{m}}$).

Remark 2. The ORMs (3) have the same matrices as the leader (2), but should not be confused with the leader itself. The difference is that in (3), r_i is to be designed. One can think about (3) as a virtual leader for each agent in (1). Accordingly, Scenario I would be solved when all these virtual leaders attain consensus, whereas Scenario II would be solved when all these virtual leaders track the actual leader (2).

The following lemma is a direct application of the KYP lemma (Khalil, 2002, Lemma 6.3).

Lemma 1. Under Assumption 2, there exist matrices $P > 0$, and $Q \triangleq R^T R + \epsilon P > 0$ for some matrix R and constant $\epsilon > 0$, such that

$$A_{\mathrm{m}}^T P + P A_{\mathrm{m}} = -Q,\ PB = C. \tag{5}$$

4. D-MRACO FOR CONSENSUS AND TRACKING

Consider the decentralized MRACO controller (D-MRACO)

$$\begin{aligned} u_i &= K_i^T(t)\hat{x}_i + L_i^T(t) r_i \\ \dot{K}_i &= -\Gamma_k \hat{x}_i(\epsilon_{i,\mathrm{m}}^T + \epsilon_{i,\mathrm{o}}^T) \\ \dot{L}_i &= -\Gamma_l r_i(\epsilon_{i,\mathrm{m}}^T + \epsilon_{i,\mathrm{o}}^T) \end{aligned} \tag{6}$$

where $\epsilon_{i,\mathrm{m}} = y_i - y_i^{\mathrm{m}}$, $\epsilon_{i,\mathrm{o}} = y_i - \hat{y}_i$, and Γ_k, Γ_l are arbitrary positive definite matrices. The time-varying gain matrices $K_i(t)$ and $L_i(t)$ are the estimates of the ideal matching gains K^* and $L^* \triangleq \Lambda^{-1}$, respectively, by agent i. Then, the following proposition can be inferred by (Ohrem and Holden, 2021, Theorem 1).

Proposition 1. Under Assumptions 1-3, consider each agent $i \in \mathcal{V}$ in (1) with the corresponding ORM (3) and CRM (4). Design the observer gain in (4) as $F = -\rho B$ with $\rho > 0$ such that

$$M \triangleq \begin{pmatrix} Q & -C\Lambda K^{*T} \\ -K^*\Lambda C^T & S \end{pmatrix} > 0 \tag{7}$$

where $S = Q + 2\rho CC^T - C\Lambda K^{*T} - K^*\Lambda C^T$ and Q in defined in Lemma 1. Then, when r_i is bounded, decentralized tracking can be realized under the D-MRACO (6), i.e., $\lim_{t\to\infty} \|x_i(t) - x_i^{\mathrm{m}}(t)\| = 0$. Moreover, the adaptive gains $K_i(t)$ and $L_i(t)$ converge to some matrices with finite elements.

Remark 3. Since ΛK^{*T} is unknown, LMI (7) can only be guaranteed from a robustness point of view. In other words, for a given A_{m}, one should find a ρ such that $M > 0$ for any point inside the known set $\mathcal{D}$. However, whether such a ρ exists is not guaranteed, and can only be checked numerically, which implies that $\mathcal{D}$ may not be arbitrarily large Ohrem and Holden (2021). In summary, the knowledge of $\mathcal{D}$ is needed in practice to verify LMI (7) (see Section 5).

In the following, we address the proper design of r_i in order to solve Scenarios I and II.

4.1 Leaderless Synchronization

The agents have access to the output of their neighbors according to the communication graph $\mathcal{G}$. The design of r_i for the ORMs (3) is the result of an observer-type distributed consensus protocol

$$\begin{aligned} \dot{v}_i &= (A_{\mathrm{m}} + BK_{\mathrm{m}})v_i + F_{\mathrm{m}}\Big(\gamma \sum_{j=1}^{N} a_{ij} C^T(v_i - v_j) - \delta_i\Big) \\ r_i &= K_{\mathrm{m}} v_i \end{aligned} \tag{8}$$

where v_i is an internal state and $\delta_i = \gamma \sum_{j=1}^{N} a_{ij}(y_i^{\mathrm{m}} - y_j^{\mathrm{m}})$. The matrix gains $K_{\mathrm{m}}, F_{\mathrm{m}}$ are designed such that $A_{\mathrm{m}} + BK_{\mathrm{m}}$ is Hurwitz and $F_{\mathrm{m}} = -P_{\mathrm{m}}^{-1}C$ where $P_{\mathrm{m}} > 0$ is a solution to

$$A_{\mathrm{m}}^T P_{\mathrm{m}} + P_{\mathrm{m}} A_{\mathrm{m}} - 2CC^T < 0. \tag{9}$$

The scalar γ represents the coupling gain inside the network.

We have the following theorem for Scenario I.

Theorem 1. Under Assumptions 1-3, the LMI (7) and $\gamma \geq \frac{1}{\Re(\lambda_2(\mathcal{L}))}$, D-MRACO (6) with ORMs (3), CRMs (4) along with distributed input (8) solve Scenario I.

Proof. First, note that for each agent $i \in \mathcal{V}$, the evolution of the ORM (3) is independent of the evolution of agent (1) itself. The proof can thus be divided into two parts, i.e., the distributed consensus over the ORMs ($x_i^{\mathrm{m}} \to x_j^{\mathrm{m}}$) and the decentralized tracking of the agents to the corresponding ORM ($x_i \to x_i^{\mathrm{m}}$).

Let us define $\xi_i = (x_i^{\mathrm{m}T}, v_i^T)^T$ and

$$\overline{A_{\mathrm{m}}} = \begin{pmatrix} A_{\mathrm{m}} & BK_{\mathrm{m}} \\ 0 & A_{\mathrm{m}} + BK_{\mathrm{m}} \end{pmatrix},\ \mathcal{H} = \begin{pmatrix} 0 & 0 \\ -F_{\mathrm{m}}C^T & F_{\mathrm{m}}C^T \end{pmatrix}. \tag{10}$$

Then, it follows from (3) and (8) that

$$\dot{\xi}_i = \overline{A_{\mathrm{m}}}\xi_i + \gamma \sum_{j=1}^{N} \mathcal{L}_{ij}\mathcal{H}\xi_j, \qquad j = 1, \cdots, N. \tag{11}$$

In fact, (11) is a standard output feedback consensus dynamics that has been studied in the literature Li et al. (2010). Note, for any $i \in \{2, \cdots, N\}$,

$$\begin{aligned} & P_{\mathrm{m}}(A_{\mathrm{m}} + \gamma\lambda_i(\mathcal{L})F_{\mathrm{m}}C^T) + (A_{\mathrm{m}} + \gamma\lambda_i(\mathcal{L})F_{\mathrm{m}}C^T)^* P_{\mathrm{m}} \\ =& P_{\mathrm{m}}A_{\mathrm{m}} + A_{\mathrm{m}}^T P_{\mathrm{m}} - \gamma\lambda_i(\mathcal{L})CC^T - \gamma\lambda_i^*(\mathcal{L})CC^T \\ =& P_{\mathrm{m}}A_{\mathrm{m}} + A_{\mathrm{m}}^T P_{\mathrm{m}} - 2\gamma\Re(\lambda_i(\mathcal{L}))CC^T \\ \leq& P_{\mathrm{m}}A_{\mathrm{m}} + A_{\mathrm{m}}^T P_{\mathrm{m}} - 2CC^T < 0. \end{aligned} \tag{12}$$

where we have used $F_{\mathrm{m}} = -P_{\mathrm{m}}^{-1}C$, $\gamma \geq \frac{1}{\Re(\lambda_2(\mathcal{L}))}$, and the LMI (9). This implies that $A_{\mathrm{m}} + \gamma\lambda_i(\mathcal{L})F_{\mathrm{m}}C^T$ are Hurwitz for all $i = 2, \cdots, N$. According to (Li et al., 2010, Theorems 1-2). The input r_i (8) solves the consensus problem of ξ_i, i.e., $\xi_i(t) \to \xi_j(t)$. In particular, $x_i^{\mathrm{m}}(t) \to x_j^{\mathrm{m}}(t)$ and $v_i(t) \to 0$ for all $i, j \in \mathcal{V}$.

Since $v_i(t) \to 0$, we know from (8) that r_i is bounded and $r_i(t) \to 0$ for all $i \in \mathcal{V}$. By Proposition 1, decentralized tracking can be realized under the D-MRACO (6), i.e., $x_i(t) \to x_i^{\mathrm{m}}(t)$.

Since $x_i^{\mathrm{m}}(t) \to x_j^{\mathrm{m}}(t)$ and $x_i(t) \to x_i^{\mathrm{m}}(t)$, it is guaranteed that $x_i(t) \to x_j(t)$ for any $i, j \in \mathcal{V}$ as $t \to \infty$. This completes the proof. ∎

4.2 Leader-follower Tracking

Let us define a matrix $\hat{\mathcal{L}} = \mathcal{L} + \mathcal{N}_L$ where $\mathcal{N}_L = \mathrm{diag}(a_{10}, a_{20}, \cdots, a_{N0})$. It is known that all the eigenvalues of $\hat{\mathcal{L}}$ have positive real parts Li et al. (2010).

In line with Li et al. (2010), it is assumed that, for the leader (2), only a subset of followers have access to the output information y_0 while all the followers have access to the external input r_0. An observer-type distributed tracking protocol can be designed as

$$\begin{aligned}\dot{v}_i =& (A_{\mathrm{m}} + BK_{\mathrm{m}})v_i \\ &+ F_{\mathrm{m}}\Big(\gamma \sum_{j=1}^{N} a_{ij} C^T(v_i - v_j) + \gamma a_{i0} C^T(v_i - v_0) - \zeta_i\Big) \\ r_i =& K_{\mathrm{m}} v_i + r_0 \end{aligned} \tag{13}$$

where v_0 is the state of the system $\dot{v}_0 = (A_{\mathrm{m}} + BK_{\mathrm{m}})v_0$ and

$$\zeta_i = \gamma \sum_{j=1}^{N} a_{ij}(y_i^{\mathrm{m}} - y_j^{\mathrm{m}}) + \gamma a_{i0}(y_i^{\mathrm{m}} - y_0). \tag{14}$$

The matrix gains $K_{\mathrm{m}}, F_{\mathrm{m}}$ are designed as in (8).

We have the following theorem for Scenario II.

Theorem 2. Under Assumptions 1-3, the LMI (7) and $\gamma \geq \frac{1}{\Re(\lambda_1(\hat{\mathcal{L}}))}$, D-MRACO (6) with ORMs (3), CRMs (4) along with distributed input (13) solve Scenario II.

Proof. The proof can be conducted following similar procedures as the proof of Theorem 1, in combination with (Li et al., 2010, Corollary 4). Thus it can be omitted. We remark that as $v_i(t) \to 0$, it follows from (13) that $r_i \to r_0$, thus r_i is bounded since r_0 is bounded by specification (it will be shown in Section 5 that D-MRACO can be robust when the boundness of r_0 is violated). ∎

Several remarks are readily resented in order.

Remark 4. The proposed framework D-MRACO contains two types of observers: the decentralized state observers $\hat{x}_i$ (the CRMs (4)) and the distributed consensus observers v_i (see (8) for Scenario I and (13) for Scenario II). This double observer design allows for solving Scenario I and II consistently based on output information (i.e. without feedback from x_i), in a unified way.

Remark 5. It is worth mentioning that as compared to MRAC-based approaches over undirected communication graphs, see e.g. Yue et al. (2021); Yue et al. (2020); Baldi and Frasca (2019); Azzollini et al. (2020); Goel and Roy (2021); Goel et al. (2022), the proposed approach uses the more relaxed and standard directed spanning tree assumption, in line with multiagent systems literature Li et al. (2010); Mei et al. (2021); Yue et al. (2021).

Remark 6. Some differences and contributions are worth remarking with respect to standard approaches in the literature under the directed spanning tree assumption, namely MRACon in Mei et al. (2021) and the classical results in Li et al. (2010). MRACon is state feedback and applies to leaderless synchronization of agents with known open-loop dynamics A and matched uncertainties; on the other hand, D-MRACO is output feedback and addresses unknown A and both leaderless and leader-follower cases. Meanwhile, the results in Li et al. (2010) do not allow uncertainties in the open-loop dynamics A of the agents, whereas the proposed D-MRACO is designed against such uncertainties.

5. NUMERICAL EXAMPLES

Consider a network of $N = 6$ second-order agents (1) with

$$A = \begin{pmatrix} -0.5 & 0.05 \\ 1 & 0 \end{pmatrix}, \ B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ \Lambda = 2, \ C = \begin{pmatrix} 1 \\ 0.2 \end{pmatrix}. \tag{15}$$

The true values of A and Λ are unknown for control design. Note that the agents are open-loop unstable.

Let us choose A_{m} as

$$A_{\mathrm{m}} = \begin{pmatrix} -0.6 & -0.05 \\ 1 & 0 \end{pmatrix}. \tag{16}$$

It can be verified that Assumption 1 holds with $\Lambda K^{*T} = (-0.1, -0.1)$. This true value of ΛK^{*T} is also unknown for control design. However, it is known that $\Lambda K^{*T} \in \mathcal{D} \triangleq \{(x, y) \in \mathbb{R}^2 | -0.12 \leq x \leq 0.12, -0.1 \leq y \leq 0.1\}$. It can be further verified that Assumptions 2-3 hold. In fact, $G_{\mathrm{m}}(s) = \frac{s+0.2}{s^2+0.6s+0.05}$ and it is SPR since $\Re(G_{\mathrm{m}}(j\omega)) = \frac{0.4\omega^2+0.01}{(0.05-\omega^2)^2+0.36\omega^2} > 0$, $\forall\omega$, and $\lim_{|\omega|\to\infty} \omega^2 \Re(G_{\mathrm{m}}(j\omega)) = 0.4 > 0$ (Ioannou and Sun, 2012, Theorem 3.5.1). Note, again, the transfer function of the real plant is not even PR (since A is unstable), which is very different from $G_{\mathrm{m}}(s)$.

To design a suitable observer gain in (4), we need to find a ρ such that the LMI (7) holds for any point ΛK^{*T} inside $\mathcal{D}$, as explained in Remark 3. For any candidate ρ, verifying LMI (7) numerically on a discrete grid in $\mathcal{D}$ is sufficient since the LMI constraint is convex. This process is accomplished with CVX toobox in Matlab Grant and Boyd (2014). The minimum value ρ found to be valid is $\rho = 13.181$. Let us select $\rho = 20$. In this case, the matrix P and Q in Lemma 1 can be chosen as $P = \begin{pmatrix} 1 & 0.2 \\ 0.2 & 0.1794 \end{pmatrix}$ and $Q = \begin{pmatrix} 0.8 & -0.0094 \\ -0.0094 & 0.02 \end{pmatrix}$. In other words, this means that with A_{m} in (16) and $\rho = 20$, the proposed method is guaranteed to be valid for any real system $A = \begin{pmatrix} a_1 & a_0 \\ 1 & 0 \end{pmatrix}$ where $a_1 \in [-0.72, -0.48]$ and $a_0 \in [-0.15, 0.05]$.

In the simulations, the initial states of the variables $x_i, x_i^{\mathrm{m}}, \hat{x}_i, v_i$ (and x_0, v_0 in Scenario II) are randomly chosen following the normal Gaussian distribution. The initial adaptive gains K_i, L_i are set as zero and the constant gains $\Gamma_k = I_2$ and $\Gamma_l = 2$.

Scenario I: Leaderless Synchronization. The communication topology among the agents is a directed ring with edge set $\mathcal{E} = \{e_{12}, e_{23}, e_{34}, e_{45}, e_{56}, e_{61}\}$: in this case, $\Re(\lambda_2(\mathcal{L})) = 0.5$, leading to the lower bound of the coupling gain $\gamma \geq 2$. Let us select $\gamma = 2$. The profiles of the states of the agents and the adaptive gains are shown in Fig. 1-2. It can be seen that the agents reach consensus and the adaptive gains converge to bounded values. In particular, we know that the final consensus value is zero, i.e., the agents are all stabilized, since the SPR property of the ORM (3) requires A_{m} to be stable.

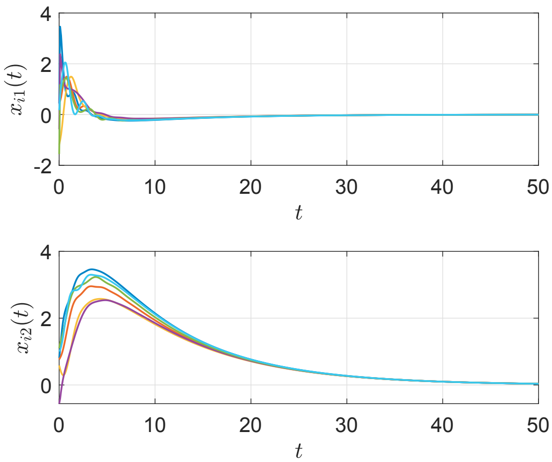

Fig. 1. Leaderless Synchronization: States of the agents x_i.

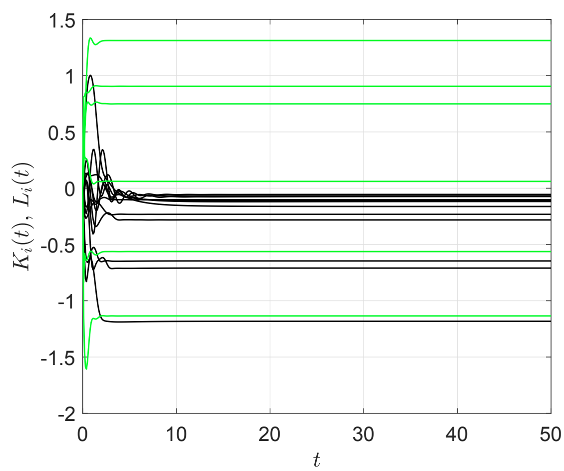

Fig. 2. Leaderless Synchronization: Adaptive gains of the agents K_i (element-wise, in black) and L_i (in green).

Scenario II: Leader-follower Tracking. The communication topology among the agents is the same ring as above, and the leader has a single neighbor which is agent 1 (i.e., a_{10}=1 and a_{i0}=0 for $i = 2, \cdots, 6$): in this case, $\Re(\lambda_1(\hat{\mathcal{L}})) = 0.1187$, leading to the lower bound of the coupling gain $\gamma \geq 8.4226$. Let us select $\gamma = 10$. The external input for the leader is set as $r_0 = \sin(t)$. The profiles of the states of the agents and the adaptive gains are shown in Fig. 3-4, where leader-follower tracking is achieved and the adaptive gains K_i, L_i converge to the ideal ones $K^* = (-0.05, -0.05)^T$, $L^* = 0.5$, respectively. However, it should be noted that the ideal convergence of the adaptive gains is not guaranteed in general, unless the external reference input is persistently exciting, in line with classical adaptive control theory.

The last experiment shows that, in spite of the boundedness of r_0 required for theoretical guarantees, D-MRACO can be robust in a finite time period for external signal that is unbounded as t goes to infinity. Let the external

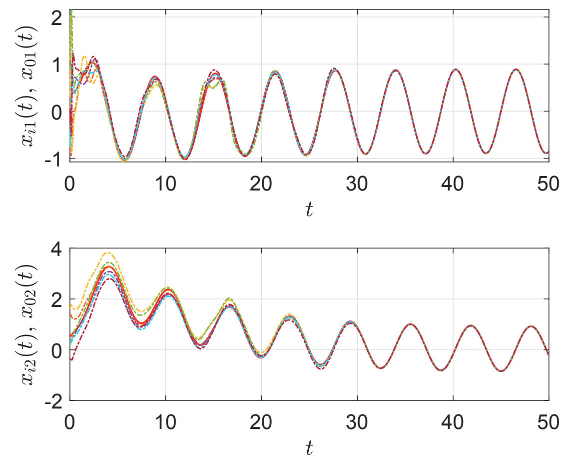

Fig. 3. Leader-follower Tracking: States of the followers x_i (dashed lines) and the leader x_0 (solid lines in red). The external input of the leader $r_0 = \sin(t)$.

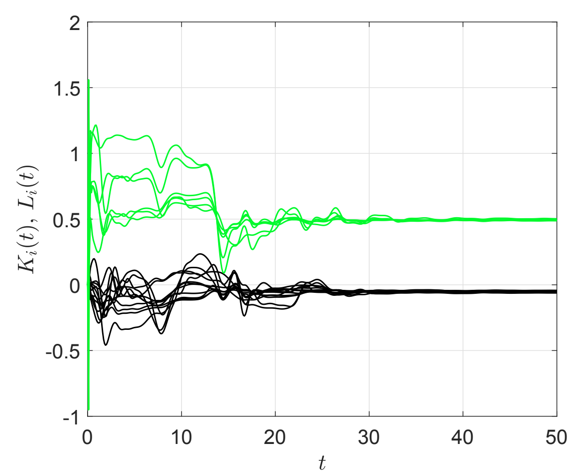

Fig. 4. Leader-follower Tracking: Adaptive gains of the followers K_i (element-wise, in black) and L_i (in green).

input for the leader be $r_0 = \sin(t) + \sqrt{t}$. Under the same other conditions as above, the profiles of the states of the agents and the adaptive gains are shown in Fig. 5-6.

6. CONCLUSIONS

We have studied the output feedback leaderless synchronization and leader-follower tracking problems for a class of unknown multiagent systems. This class of unknown systems is in line with closed-loop reference model based adaptive control literature Gibson et al. (2013, 2015); Ohrem and Holden (2021). A decentralized model reference adaptive controller and observer design (D-MRACO) is proposed to address the leaderless synchronization and leader-follower tracking problems in a unified way. Future works may include characterizing persistent excitation conditions that ensure exact ideal convergence of the adaptive gains, and addressing the cases when only a subset or none of the followers have access to the external input r_0 of the leader.

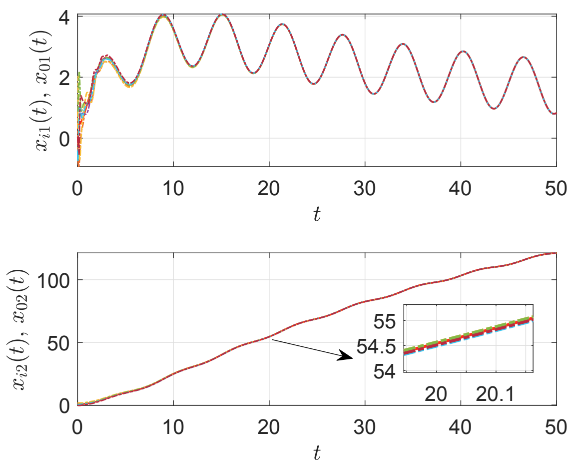

Fig. 5. Leader-follower Tracking: States of the followers x_i (dashed lines) and the leader x_0 (solid lines in red). The external input of the leader $r_0 = \sin(t) + \sqrt{t}$.

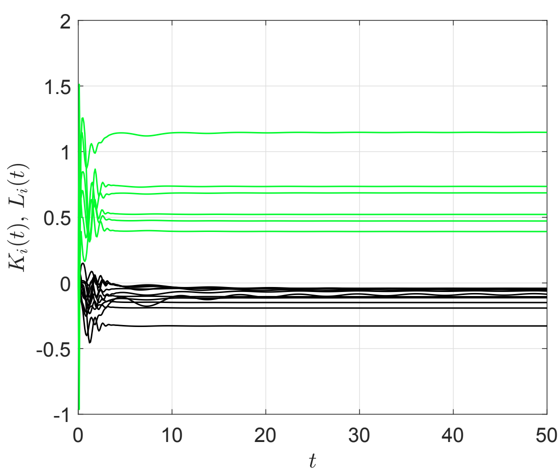

Fig. 6. Leader-follower Tracking: Adaptive gains of the followers K_i (element-wise, in black) and L_i (in green).

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