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Neuro-adaptive consensus strategy for a class of nonlinear time-delay multi-agent systems with an unmeasurable high-dimensional leader

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Dongdong Yue[1], Jinde Cao[2] ✉, Qi Li[1], Xinli Shi[1]

[1]*School of Automation, and Key Laboratory of Measurement and Control of Complex Systems of Engineering, Ministry of Education, Southeast University, Nanjing 210096, People's Republic of China*

[2]*School of Mathematics, and Research Center for complex Systems and Network Sciences, Southeast University, Nanjing 210096, People's Republic of China*

✉ *E-mail: jdcao@seu.edu.cn*

Abstract: Distributed cooperative consensus tracking problem for a class of uncertain multi-agent systems with a high-dimensional leader under a directed communication topology is concerned. Compared with related works, the dynamics of the leader agent is allowed to be different from those of the followers and may not be measured. Meanwhile, the dynamics of each follower is subject to un-modelled dynamics, unknown time-varying delays, as well as external disturbances, which makes the model more suitable in various practical applications. Based on the M-matrix and Lyapunov–Krasovskii functional method, a distributed robust radial basis function neural network controller as well as a local observer is designed for each follower so as to guarantee the ultimate boundness of the tracking errors to the leader's output signals. By appropriately cutting down the neural network parameters to be updated online, the computational burden can be greatly reduced. The effectiveness of the proposed consensus approach is testified via a numerical example.

1 Introduction

The last few decades have witnessed rapid development of the coordination control theory of multi-agent systems (MASs) due to its broad and potential applications in power systems [1], traffic control [2], robotics [3], unmanned air vehicle [4], economic dispatch and optimisation [5] etc. Basically, the study of coordinative control of MASs is devoted to analyse the emergence of various global behaviours, such as consensus [6], flocking [7], and formation [8], as a result of local interactions among the individuals.

The consensus is generally concerned with how to reach an agreement among a group, e.g. the positions of dynamical oscillators, the velocities of mobile vehicles, the angles of mechanical arms, and so on. This fundamental yet critical coordinative behaviour has drawn much attention in both scientific area and real engineering applications, see recent survey papers [9, 10] and references therein. The consensus in MASs can be roughly divided into two cases, i.e. leaderless consensus and leader-following consensus. The latter case, also known as the consensus tracking problem, provides a reference signal for followers to track asymptotically, whereas agents in the former case synchronise to a prior unknown agreement without references. An alternative view for the leaderless consensus is that a virtual leader is introduced to provide a reference signal for all agents to track and follow [11]. Note that another research hotpot close to the leader-following consensus of MASs is the pinning synchronisation of complex networks, see [12, 13].

The consideration of uncertainties has always been one of the major topics in control theory considering technical limitations in real systems. The time delay problem is one typical and important issue which has been studied in the consensus of the MASs [14–19]. According to the components where delay exists, three different kinds of time delays are mainly concerned, i.e. state delay [16], input delay [17, 19], and communication delay [15, 18]. In [14], the consensus of the linear discrete MASs subject to diverse input and communication delays has been studied by means of frequency-domain analysis, and several communication-delay independent sufficient conditions have been derived. However, most of the above works are performed on linear or simply coupled MASs. Consensus tracking of a class of nonlinear stochastic MASs under impulsive control has been studied in [16], where the state delays of agents are assumed to be time-varying. Moreover, when referring to the uncertain dynamics of agents, neural network (NN)-based protocols have become very popular because of their powerful approximation capabilities [20–22]. In [20], the consensus control of a class of nonlinear time delay MASs has been studied by using radial basis function NNs (RBFNNs), where the potential singularity problem of the controller is avoided by relaxing the consensus objective to a ‘ball’ region rather than a single origin. This method has been extended to the leader-following case for second-order nonlinear MASs [21] and a class of state-delay nonlinear MASs [22], where a common salient contribution in both studies is the scalar-form adaptive law for the NN weights to alleviate the computation burden.

On the other hand, it is very difficult in many cases to measure the states of all the agents due to the sensor limits, noise perturbations or data packet dropout, thus stimulating wide discussions on the observer-based consensus protocol in MASs recently [23–28]. In [23], the consensus tracking control algorithms have been proposed based on the local observer and distributed pinning observer separately. The observer-based event-triggered consensus control scheme for MASs has been developed in [27]. By designing NN-based state observers and adaptive backstepping controllers, the uniformly ultimately bounded consensus problem for a class of high-order MASs with semi-strict feedback followers has been addressed in [24] in the presence of unmeasurable states and uncertain dynamics. For the finite time consensus of second-order MASs, discontinuous observers have been designed in [25]. Cao *et al*. [26] focus on observer design strategies considering variable topologies. A fractional-order observer has been presented for second-order MASs in [28].

It should be noticed that most of the existing consensus tracking results above focus on homogeneous MASs, in which the leader and the followers are assumed to have identical internal dynamics. However, it is much more common in practice that the leader is proactive and has different strategies, even lives in different dimensions inside a group. The heterogeneous consensus, as an emerging research topic that takes dynamics of different agents into consideration, has drawn much attention in recent years [29]. Nevertheless, the consensus tracking problem of MASs with a heterogeneous leader, particularly with a higher-dimensional one, does not draw enough attention [30]. In [31], the consensus tracking with a single high-dimensional leader modelled by a general form of linear systems is considered through the distributed control method, where the followers are simply coupling agents without inherent dynamics. Recently, by using the neuro-adaptive method, the consensus tracking problem of a class of uncertain MASs with a single high-dimensional dynamic leader has been considered in [32], where the followers asymptotically track the leader's output signals with theoretically bounded residual errors. However, time delays are absent in these works.

Inspired by the above discussions, we address a consensus tracking problem for a class of heterogeneous nonlinear MASs, in which the leader is described as a determinate Lipschitz-type nonlinear system with higher-dimensional states, while the followers are dynamical systems subject to partially un-modelled dynamics as well as unknown time-varying delays. To our best knowledge, the observer-based consensus control problem of the heterogeneous nonlinear MASs with time delay has not been well understood, which still remains a challenging task. The main contributions of this paper are listed as follows.

i. To the best of the authors' knowledge, time delays are firstly considered in the consensus tracking problem with a high-dimensional leader.
ii. The influences of unknown time-varying delays of agents are neutralised by employing an appropriate Lyapunov–Krasovskii functional. Compared with the related existing results, the unknown time delays are extended to be time-varying.
iii. The un-modelled dynamics of agents is approximated online with RBFNNs. Furthermore, the method of designing NN weight adaptive law into scalar forms in [21, 22] is used here to alleviate the computation burden.

The remaining part of this paper is organised as follows. After some notations are illuminated later, Section 2 presents some preliminaries and describes the problem. The main theoretical results of this paper are formulated in Section 3. Then a numerical example is provided in Section 4. Section 5 finally gives the conclusion of the current paper and interests for future works.

Throughout this paper, $\mathbb{N}$ ($\mathbb{N}^*$) is the set of natural numbers (excluded 0). $\mathbb{R}$, $\mathbb{R}^n$, $\mathbb{R}^{n\times p}$ denote the sets of real numbers, n dimensional real vectors, $n\times p$ real matrices, respectively. Let $\mathbf{O}_n$ ($\boldsymbol{I}_n$) denote the $n\times n$ zero (identity) matrix, and $\mathbf{0}_n(\mathbf{1}_n)$ be the n-dimensional column vector with all elements being zero (one). $\|\cdot\|$ ($\|\cdot\|_F$) denotes the Euclidian (Frobenius) norm of a vector (matrix). The eigenvalues of matrix A is denoted by $\lambda(A)$, among which $\lambda_{\min}(A)$ is the smallest one. The matrix inequality $A>0$ means that A is positive definite. AB, $A\otimes B$ represent ordinary multiplication and Kronecker product of matrices A and B, respectively. The column vectorisation of a set of components $x_1, x_2, \dots, x_n$, which could be either vectors or matrices, is denoted by $\mathrm{Col}(x_1, x_2, \dots, x_n)$, i.e. $\mathrm{Col}(x_1, x_2, \dots, x_n) = (x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \dots, x_n^{\mathrm{T}})^{\mathrm{T}}$. The term ‘iff’ is the abbreviation of ‘if and only if’.

2 Preliminaries and problem statement

2.1 Graph theory

Let $\mathscr{G}(\mathscr{V}, \mathscr{E}, \mathscr{A})$ or $\mathscr{G}$, if no confusion will arise, denote a weighted digraph with the vertices $\mathscr{V} = \{1, 2, \dots, N\}$, the directed edges $\mathscr{E} \subseteq \{(i,j), i, j \in \mathscr{V}\}$, and a weighted adjacency matrix $\mathscr{A} = (a_{ij}) \in \mathbb{R}^{N\times N}$. We define $a_{ii} = 0$, and $a_{ij} > 0$, $i \neq j$ iff $(j, i) \in \mathscr{E}$, which means that there exists an edge from vertex j to i. A path on $\mathscr{G}$ from vertex v_1 to vertex v_s corresponds a sequence of ordered edges of the form (v_k, v_{k+1}), $k = 1, 2, \dots, s-1$. A digraph is said to contain a directed spanning tree, if there exists a node called root, from which one can find a path to every other node. The Laplacian matrix $\mathscr{L} = (l_{ij}) \in \mathbb{R}^{N\times N}$ of $\mathscr{G}$ is defined as: $l_{ij} = -a_{ij}$, $i\neq j$, and $l_{ii} = \sum_{k=1, k\neq i}^{N} a_{ik}$, $i = 1, 2, \dots, N$. In this study, $\mathscr{G}$ is regarded as a simple graph, for which multiple edges and self-loops are not permitted.

2.2 RBFNN and function approximation

NNs have been widely studied and applied due to their powerful ability of approximation, among which multi-layer perceptron (MLP) and RBFNN are outstanding representatives. In this work, we prefer RBFNN to get close to the unknown nonlinear functions considering its typical neurologic characteristics, e.g. ‘near-excitation and far-inhibition’. It is known that any continuous function $\varphi(x)\colon \mathbb{R}^m \mapsto \mathbb{R}^n$ can be approximated by RBFNN as

$$\tilde{\varphi}(x) = W^{\mathrm{T}} S(x),$$

where $x \in \Omega_x \subset \mathbb{R}^m$, Ω_x is a compact set and $W \in \mathbb{R}^{q\times n}$ is the adjustable NN weight matrix. $S(x) = (s_1(x), s_2(x), \dots, s_q(x))^{\mathrm{T}}$ with Gaussian basis functions $s_i(x) = \exp(-[((x-v_i)^{\mathrm{T}}(x-v_i))/b_i^2])$, $i = 1, 2, \dots, q$, in which $v_i = (v_{i1}, v_{i2}, \dots, v_{im})^{\mathrm{T}} \in \mathbb{R}^m$, $b_i \in \mathbb{R}$ are the centre and width of the response area of hidden neuron i, respectively. q denotes the number of hidden neurons.

The *universal approximation theorem* [33] shows that RBFNN can approximate arbitrary continuous function with arbitrary precision over a compact set by choosing sufficient hidden neurons. For $\varphi(x)$, there exists an ideal weight matrix $W^* \in \mathbb{R}^{q\times n}$ such that

$$\varphi(x) = W^{*\mathrm{T}} S(x) + \varepsilon(x)$$

in which $\varepsilon(x) \in \mathbb{R}^n$ is the approximation error that satisfies

$$\|\varepsilon(x)\| \leq \varepsilon, \quad \forall\, x \in \Omega_x$$

with a pre-designed error bound $\varepsilon > 0$. Mathematically, we define W^* as the value of W which minimises $\|\varepsilon(x)\|$ for all $x \in \Omega_x$:

$$W^* := \arg \min_{W \in \mathbb{R}^{q\times n}} \left\{ \sup_{x\in\Omega_x} \|\varphi(x) - W^{\mathrm{T}} S(x)\| \right\}.$$

Generally, the ideal matrix W^* needs to be estimated and could not be directly used in the controller design.

2.3 Problem formulation

The concerned MAS consists of $N+1$ agents in which agent 0 serves as the leader and agent $1 \sim N$ are regarded as followers. The dynamic evolution of the leader agent is as follows:

$$\begin{cases} \dot{x}_0(t) = \hat{A} x_0(t) + g(x_0(t), t), \\ y(t) = \hat{C} x_0(t), \end{cases} \tag{1}$$

where $x_0(t) \in \mathbb{R}^{nm}$, $n, m \in \mathbb{N}^*$, indicate the leader's state at time t, $\hat{A} \in \mathbb{R}^{nm\times nm}$ describes the known linear part dynamics of the system, nonlinear vector-valued function $g(x_0(t), t)\colon \mathbb{R}^{nm} \times [0, +\infty) \mapsto \mathbb{R}^m$ is assumed to be continuously differentiable and satisfy the following global Lipschitz condition:

Assumption 1: There exists $\psi \geq 0$, such that $\|g(x,t) - g(y,t)\| \leq \psi \|x - y\|$, $\forall x, y \in \mathbb{R}^{nm}$, $t > 0$.

$y(t) \in \mathbb{R}^m$ represents the output signals of the leader, and the system output matrix $\hat{C} \in \mathbb{R}^{m\times nm}$ is fixed as $\hat{C} = (\mathbf{O}_m, \mathbf{O}_m, \dots, \mathbf{O}_m, \boldsymbol{I}_m)$ for simplicity. The pair $(\hat{C}, \hat{A})$ is supposed to be detectable. Notice the state vector of the leader $x_0(t)$ can be rewritten as $x_0(t) = \mathrm{Col}(x_0^1(t), x_0^2(t), \dots, x_0^n(t))$ with

$x_0^i(t) \in \mathbb{R}^m, i = 1, 2, \ldots, n$. To formulate the consensus tracking problem, one may partition the matrix $\hat{A} = (\hat{a}_{ij}) \in \mathbb{R}^{nm \times nm}$ into

$$\begin{pmatrix} \hat{A}_{11} & \hat{A}_{12} & \cdots & \hat{A}_{1n} \\ \hat{A}_{21} & \hat{A}_{22} & \cdots & \hat{A}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{A}_{n1} & \hat{A}_{n2} & \cdots & \hat{A}_{nn} \end{pmatrix}, \tag{2}$$

where $\hat{A}_{ij} \in \mathbb{R}^{m \times m}, i, j = 1, 2, \ldots, n$. Then, the nonlinear function $g(x_0(t), t)$ is further assumed to be divided into $g(x_0(t), t) = \mathrm{Col}(g_1(x_0^1(t), t), g_2(x_0^2(t), t), \ldots, g_n(x_0^n(t), t))$, in which $g_i(x_0^i(t),\ t) : \mathbb{R}^m \times [0, +\infty) \mapsto \mathbb{R}^m$ is naturally a continuously differentiable vector function.

The dynamics of the ith $(1 \le i \le N)$ follower is described as

$$\begin{aligned} \dot{x}_i(t) = Ax_i(t) + f_i(x_i(t)) + h_i(x_i(t - \tau_i(t))) \\ + d_i(t) + u_i(t), \end{aligned} \tag{3}$$

where $x_i(t) \in \mathbb{R}^m$ is the state vector, $A \in \mathbb{R}^{m \times m}$ is a known matrix depicting the available linear dynamics. $f_i(x_i(t)), h_i(x_i(t)) : \mathbb{R}^m \mapsto \mathbb{R}^m$ are unknown but intrinsically continuous vector functions representing the nonlinear behaviours. $\tau_i(t)$ and $d_i(t)$ are separately the unknown time-varying delay and external disturbances.

Remark 1: Note that in some previous works on the consensus problem of nonlinear MASs [20, 22], the state delays of the agents are assumed as time-invariant. In this study, we consider that since the state delays are intrinsically unknown, it is more practical to assume them as time-varying. Besides, the dynamics of the followers is partially unknown here as in [32] since agents with completely unknown or, on the contrary, completely known dynamics are not common in realistic systems. In many cases, the linear dynamics of agents is available while the nonlinear motions are unclear, under which circumstances the model considered here is applicable.

The communication topology among the N followers is described as $\mathscr{G}(\mathscr{V}, \mathscr{E}, \mathscr{A})$. Similarly, we use $\bar{\mathscr{G}}(\bar{\mathscr{V}}, \bar{\mathscr{E}}, \bar{\mathscr{A}})$ to represent the topology involving the leader 0. Since the leader always serves to issue the command, it is reasonable to assume that the behaviours of the leader will not be affected by those of the followers and only leader's output information can be acquired by a part of followers. Moreover, the initial state of the leader $x_0(0)$ is assumed as unknown to all of the followers for practical considerations. Accordingly, the Laplacian matrix of $\bar{\mathscr{G}}$ can be divided into the following:

$$\bar{\mathscr{L}} = \begin{pmatrix} 0 & \mathbf{0}_N^{\mathrm{T}} \\ -\mathbf{a}_0 & \tilde{\mathscr{L}} \end{pmatrix}, \tag{4}$$

where $\tilde{\mathscr{L}} = \mathscr{L} + \mathrm{diag}(\mathbf{a}_0)$, $\mathbf{a}_0 = (a_{10}, a_{20}, \ldots, a_{N0})^{\mathrm{T}}$, and $a_{i0} > 0$ iff follower i is directly pinned by the leader.

Assumption 2: Digraph $\bar{\mathscr{G}}$ contains a spanning tree which is rooted at the leader node 0.

The control objective of this study is to design consensus tracking protocols $u_i(t), i = 1, 2, \ldots, N$, under which all followers can ultimately synchronise to the leader's output signals with bounded errors described in the following definition.

Definition 1: The MAS composing of the high-dimensional leader (1) and followers (3) is said to solve the consensus tracking problem with positive residual error ϖ, if $\lim_{t \to \infty} \| x_i(t) - y(t) \| \le \varpi, \forall i = 1, 2, \ldots, N$ [32].

It should be pointed out that the main challenges in this control task are twofold. First, the unmeasurable dynamics of the high-dimensional leader needs to be reconstructed for each follower by designing distributed observers. Second, the effects of unmodelled dynamics, unknown time-varying delays as well as unknown disturbances need to be coped with online while only neighbouring agents' information is available. To proceed, the following standard assumptions are needed.

Assumption 3: The unknown function $h_i(x_i(t))$ satisfies the inequalities $\| h_i(x_i(t)) \| \le \rho_i(x_i(t)),\ i = 1, 2, \ldots, N$, where $\rho_i(\cdot)$ are known positive smooth functions [34].

Assumption 4: The unknown disturbances are bounded, i.e. $\| d_i(t) \| \le d_M, \forall i = 1, 2, \ldots, N$ with a positive scalar d_M.

Assumption 5: The unknown time delay $\tau_i(t)$ satisfies that $\dot{\tau}_i(t) \le \tau_d < 1,\ 0 \le \tau_i(t) \le \tau_{\max},\ i = 1, 2, \ldots, N$ with known constants τ_d and $\tau_{\max}$.

Before moving on, we state the following two lemmas to be used in later parts.

Lemma 1: The Laplacian matrix $\mathscr{L}$ of a digraph $\mathscr{G}$ has a single eigenvalue 0 and leaves all the other eigenvalues located in the right half plane of the complex coordinate system iff $\mathscr{G}$ has a spanning tree [35].

Lemma 2: For a non-singular matrix $A \in \mathbb{R}^{n \times n}$, the following statements are equivalent [36]:

1. A is an M matrix.
2. All the non-diagonal elements of A are non-positive and all the elements of A^{-1} are non-negative.
3. All the eigenvalues of A have positive real parts, i.e. $\mathfrak{R}(\lambda_i(A)) > 0,\ i = 1, 2, \ldots, n$.
4. There exists a diagonal matrix $\Theta > 0$ such that $\Theta A + A^{\mathrm{T}}\Theta > 0$.

3 Main results

3.1 Distributed observer design

To be sensitive to the consensus tracking errors, each follower must maintain an observer to figure out the unmeasurable dynamics of the leader. In this subsection, a distributed observer design strategy based on local information is presented.

Under Assumption 2, we can conclude that $\tilde{\mathscr{L}}$ is an M-matrix from Lemma 1. Immediately, there exist a positive diagonal matrix

$$\Xi = \mathrm{diag}(\xi_1, \xi_2, \ldots, \xi_N), \tag{5}$$

such that $\Xi\tilde{\mathscr{L}} + \tilde{\mathscr{L}}^{\mathrm{T}}\Xi > 0$ by Lemma 2. One feasible choice of Ξ can be obtained by letting $\xi = (\xi_1, \xi_2, \ldots, \xi_N)^{\mathrm{T}} = (\tilde{\mathscr{L}}^{\mathrm{T}})^{-1}\mathbf{1}_N$ [37]. We denote

$$\underline{\lambda} = \lambda_{\min}(\tilde{\mathscr{L}} + \Xi^{-1}\tilde{\mathscr{L}}^{\mathrm{T}}\Xi) \tag{6}$$

and it is easy to see that $\underline{\lambda}$ is a positive real number. Define $\zeta_i(t) = \mathrm{Col}(\zeta_i^1(t), \zeta_i^2(t), \ldots, \zeta_i^n(t)) \in \mathbb{R}^{nm}$ as the state of the observer maintained by follower i at time t such that

$$\begin{aligned} \dot{\zeta}_i^k(t) = g_k(\zeta_i^k(t), t) + \sum_{j=1}^{n} \hat{A}_{kj}\zeta_i^j(t) \\ -\alpha F_k \eta_i(t) \quad k = 1, 2, \ldots, n, \end{aligned} \tag{7}$$

where $\alpha > 0$ remains to be selected, $F = \mathrm{Col}(F_1, F_2, \ldots, F_n) \in \mathbb{R}^{nm \times m}$ is the feedback gain matrix to be designed, $\eta_i(t)$ is the observer-based local consensus error of follower i defined as the following:

$$\eta_i(t) = \Sigma_{j=1}^{N} a_{ij}(\zeta_j^n(t) - \zeta_i^n(t)) + a_{i0}(y(t) - \zeta_i^n(t)). \tag{8}$$

From [32], $\zeta_i(t)$ provides a full-order estimator of the leader's state for agent i.

Let $\delta_i(t) = \zeta_i(t) - x_0(t), i = 1, 2, \ldots, N$, represent the observer error system for the leader agent managed by follower i. From (1) and (7), the dynamics of $\delta_i(t)$ can be written as

$$\dot{\delta}_i(t) = g(\zeta_i(t), t) - g(x_0(t), t) + \hat{A}\delta_i(t) - \alpha F\eta_i(t) . \tag{9}$$

Then, by defining

$$\begin{aligned} \delta(t) &= \mathrm{Col}(\delta_1(t), \delta_2(t), \ldots, \delta_N(t)), \\ \zeta(t) &= \mathrm{Col}(\zeta_1(t), \zeta_2(t), \ldots, \zeta_N(t)) \text{ and} \\ \tilde{g}(\zeta(t), x_0(t), t) &= \mathrm{Col}(g(\zeta_1(t), t), g(\zeta_2(t), t), \ldots, g(\zeta_N(t), t)) \\ &\quad -\mathbf{1}_N \otimes g(x_0(t), t), \end{aligned} \tag{10}$$

one can get the matrix form of the global observer error system:

$$\dot{\delta}(t) = (\boldsymbol{I}_n \otimes \hat{A} + \alpha\tilde{\mathscr{L}} \otimes F\hat{C})\delta(t) + \tilde{g}(\zeta(t), x_0(t), t) . \tag{11}$$

The following proposition can be directly concluded from [32].

Proposition 1: Under Assumptions 1 and 2, the equilibrium point of the observer error system (11) is exponentially asymptotic stable, if there exist a positive definite matrix $P > 0 \in \mathbb{R}^{nm \times nm}$ and positive scalars $\alpha > 0$, $c_0 > 0$, such that

$$\begin{pmatrix} \hat{A}^{\mathrm{T}}P + P\hat{A} - \alpha\underline{\lambda}\hat{C}^{\mathrm{T}}\hat{C} + c_0P + \boldsymbol{I}_{nm} & \psi P \\ \psi P & -\boldsymbol{I}_{nm} \end{pmatrix} < 0. \tag{12}$$

Moreover, the feedback matrix F can be designed as

$$F = -P^{-1}\hat{C}^{\mathrm{T}} . \tag{13}$$

The above proposition provides an efficient method to design the observer so as to guarantee the convergence of $\zeta_i^n(t)$ to $y(t)$ for each follower i. It is known that a relatively larger c_0 will lead to faster converge of the observer system.

3.2 Neuro-adaptive consensus control

In this subsection, the RBFNN-based adaptive consensus tracking protocols among the followers are designed.

Denote the observer-based tracking errors for agent i as $e_i(t) = x_i(t) - \zeta_i^n(t), i = 1, 2, \ldots, N$. By (3) and (7), we have

$$\begin{aligned} \dot{e}_i(t) &= Ax_i(t) + f_i(x_i(t)) + h_i(x_i(t - \tau_i(t))) + d_i(t) + u_i(t) \\ &\quad - g_n(\zeta_i^n(t), t) - \sum_{j=1}^{n} \hat{A}_{nj}\zeta_i^j(t) + \alpha F_n\eta_i(t) . \end{aligned} \tag{14}$$

Obviously, an observe compensation strategy is necessary for further design procedure, which motivates us to divide $u_i(t)$ into the observer-based compensation term and the error-based feedback term [32] as

$$u_i(t) = u_i^{ot}(t) + u_i^{et}(t) \tag{15}$$

in which $u_i^{ot}(t)$ is designed as

$$\begin{aligned} u_i^{ot}(t) &= (\hat{A}_{nn} - A)x_i(t) + g_n(\zeta_i^n(t), t) \\ &\quad + \sum_{j=1}^{n-1} \hat{A}_{nj}\zeta_i^j(t) - \alpha F_n\eta_i(t) \end{aligned} \tag{16}$$

and $u_i^{et}(t)$ will be designed later.

Remark 2: It can be seen from (7) and (16) that in the observer and compensation design, partial dynamical information of the leader is needed to build a bridge between the high-dimensional leader and the followers.

Substituting (16) into (14) yields

$$\begin{aligned} \dot{e}_i(t) &= \hat{A}_{nn}e_i(t) + f_i(x_i(t)) + h_i(x_i(t - \tau_i(t))) \\ &\quad + d_i(t) + u_i^{et}(t) . \end{aligned} \tag{17}$$

To proceed, we first define a scalar matrix

$$V_1(t) = \frac{1}{2}\sum_{i=1}^{N} e_i(t)^{\mathrm{T}}e_i(t) . \tag{18}$$

Taking the time derivative of $V_1(t)$ along (17), we have

$$\begin{aligned} \dot{V}_1(t) &= \frac{1}{2}\sum_{i=1}^{N} e_i(t)^{\mathrm{T}}(\hat{A}_{nn} + \hat{A}_{nn}^{\mathrm{T}})e_i(t) \\ &\quad + \sum_{i=1}^{N} e_i(t)^{\mathrm{T}}\big(f_i(x_i(t)) + h_i(x_i(t - \tau_i(t))) \\ &\quad + d_i(t) + u_i^{et}(t)\big) \\ &\le \sum_{i=1}^{N} \chi_{\max} \parallel e_i(t) \parallel^2 \\ &\quad + \sum_{i=1}^{N} e_i(t)^{\mathrm{T}}\big(f_i(x_i(t)) + h_i(x_i(t - \tau_i(t))) \\ &\quad + d_i(t) + u_i^{et}(t)\big) \end{aligned} \tag{19}$$

where $\chi_{\max}$ denotes the largest real part of the eigenvalues of $\hat{A}_{nn}$. According to Assumptions 3 and 4, and Cauchy's inequality, we have

$$\begin{aligned} \dot{V}_1(t) &\le \sum_{i=1}^{N} \Big(\chi_{\max} \parallel e_i(t) \parallel^2 + e_i(t)^{\mathrm{T}}f_i(x_i(t)) \\ &\quad + e_i(t)^{\mathrm{T}}\rho_i(x_i(t - \tau_i(t))) \\ &\quad + d_M \parallel e_i(t) \parallel + e_i(t)^{\mathrm{T}}u_i^{et}(t)\Big) . \end{aligned} \tag{20}$$

Remark 3: The feedback control design cannot be accomplished directly following (20), the main challenges are twofold: first, the function $f_i(\cdot)$ and delay $\tau_i(t)$ are unknown, which further leads to the uncertainty of the known function $\rho_i(\cdot)$; second, the unknown time varying delay $\tau_i(t)$ and the timely consensus error $e_i(t)$ are essentially coupled together, which makes the control task more complicated. We need first to decouple the uncertainties $\tau_i(t)$ and $e_i(t)$ so as to deal with them separately.

Applying Young's inequality, i.e. $ab \le a^2/2 + b^2/2$, to (20), we have

$$\begin{aligned} \dot{V}_1(t) &\le \sum_{i=1}^{N} \bigg((1 + \chi_{\max}) \parallel e_i(t) \parallel^2 + e_i(t)^{\mathrm{T}}u_i^{et}(t) \\ &\quad + e_i(t)^{\mathrm{T}}f_i(x_i(t)) + \frac{\rho_i^2(x_i(t - \tau_i(t)))}{2}\bigg) \\ &\quad + \frac{Nd_M{}^2}{2} . \end{aligned} \tag{21}$$

Note that in (21), $\tau_i(t)$ and $e_i(t)$ are separated. Then, to smooth away the difficulties in the control design which are introduced by the unknown time-varying delays $\tau_i(t)$, we construct the following Lyapunov–Krasovskii functional:

$$V_2(t) = \frac{1}{2(1 - \tau_d)}\sum_{i=1}^{N} \int_{t-\tau_i(t)}^{t} \rho_i^2(x_i(s))\,\mathrm{d}s . \tag{22}$$

Taking the derivative of $V_2(t)$ gives

$$\begin{aligned}\dot{V}_2(t) &= \frac{1}{2(1-\tau_d)}\Bigg(\sum_{i=1}^{N}\rho_i^2(x_i(t)) \\ &\quad -\sum_{i=1}^{N}\rho_i^2(x_i(t-\tau_i(t)))(1-\dot{\tau}_i(t))\Bigg) \\ &\le \frac{1}{2(1-\tau_d)}\Bigg(\sum_{i=1}^{N}\rho_i^2(x_i(t)) \\ &\quad -\sum_{i=1}^{N}\rho_i^2(x_i(t-\tau_i(t)))(1-\tau_d)\Bigg) \\ &= \frac{1}{2(1-\tau_d)}\sum_{i=1}^{N}\rho_i^2(x_i(t)) - \frac{1}{2}\sum_{i=1}^{N}\rho_i^2(x_i(t-\tau_i(t))).\end{aligned} \tag{23}$$

Obviously, $V_2(t)$ is capable of compensating the uncertainties of $\tau_i(t)$. Actually, by choosing

$$V_e(t) = V_1(t) + V_2(t), \tag{24}$$

we have

$$\begin{aligned}\dot{V}_e(t) \le \sum_{i=1}^{N}\Bigg((1+\chi_{\max}) \parallel e_i(t) \parallel^2 + e_i(t)^{\mathrm{T}}u_i^{et}(t) \\ + e_i(t)^{\mathrm{T}}f_i(x_i(t)) + \frac{\rho_i^2(x_i(t))}{2(1-\tau_d)}\Bigg) + \frac{Nd_M^2}{2}.\end{aligned} \tag{25}$$

For the sake of brevity, we omit the time stamp t inside terms $x_i(t)$, $e_i(t)$ in the absence of ambiguity below.

Now the only obstacle left will be the completely unknown system functions $f_i(x_i)$. Let $x_i \in \Omega_{x_i} \subset \mathbb{R}^m$, Ω_{x_i} be a compact set. Define $\Omega_{r_i}, \Omega_{r_i}^o \subset \Omega_{x_i}$, $i = 1, 2, \ldots, N$, as

$$\Omega_{r_i} := \{x_i \big| \parallel e_i \parallel \le r_i\}, \tag{26}$$

$$\Omega_{r_i}^o := \Omega_{x_i} - \Omega_{r_i}, \tag{27}$$

where r_i is an arbitrarily small constant. Then $\Omega_{r_i}^o$ is also a compact set [34]. Since $f_i(x_i)$ is continuous on the compact set $\Omega_{r_i}^o \subset \mathbb{R}^m$, it can be well approximated by RBFNN as

$$f_i(x_i) = W_i^{*\mathrm{T}}S_i(x_i) + \varepsilon_i(x_i), \tag{28}$$

where $W_i^* \in \mathbb{R}^{q_i \times m}$ is the ideal weight matrix, $S_i(\cdot) \in \mathbb{R}^{q_i}$ includes the basis functions, q_i is the pre-designed number of neurons and $\varepsilon_i(x_i) \in \mathbb{R}^m$ is the approximation error defined as above. Then there exists a positive scalar ε_M such that

$$\parallel \varepsilon_i(x_i) \parallel \le \varepsilon_M. \tag{29}$$

We denote $w_i^* = \parallel W_i^* \parallel_{\mathrm{F}}$. Then, the adaptive consensus error-based term and the adaptive law for follower i are designed as

$$u_i^{et}(t) = \begin{cases} -\beta_i(t)e_i - \tilde{w}_i(t) \parallel S_i(x_i) \parallel^2 e_i \\ -\dfrac{1}{2(1-\tau_d)}e_i^{-1}\rho_i^2(x_i), & x_i \in \Omega_{r_i}^o, \\ 0, & x_i \in \Omega_{r_i}, \end{cases} \tag{30}$$

$$\dot{\tilde{w}}_i(t) = \Gamma_i(\parallel S_i(x_i) \parallel^2 \parallel e_i \parallel^2 - c_i\tilde{w}_i(t)), \tag{31}$$

where $\tilde{w}_i(t)$ is the estimation of the unknown ideal matrix w_i^*, $\Gamma_i, c_i > 0$ are design constants, and e_i^{-1} denotes $e_i / \parallel e_i \parallel^2$, which satisfies that $e_i^{\mathrm{T}}e_i^{-1} = (e_i^{-1})^{\mathrm{T}}e_i = 1$. $\beta_i(t)$ is a time-varying parameter to be determined later.

Remark 4: Since the term e_i^{-1} is not well defined at $e_i = \mathbf{0}_m$, the controller may suffer the singularity problem at the zero point of the error system, where the observer-based consensus goal is achieved. Once consensus is reached, it is supposed that the error feedback control action should not take power consumptions anymore. To this end, we relax the consensus purpose to a 'ball' area instead of a single origin [38].

Remark 5: For the purpose of improving the approximation precision to the unknown nonlinear function, most existing neuro-adaptive consensus control methods require a relatively large number of neurons and update every single component of the weight matrix for each agent, which will greatly increase the online computing burden. In the proposed scheme, however, it can be found that only a scalar variable, i.e. the Frobenius norm of the corresponding NN weight matrix, is updated online for each agent, which will undoubtedly reduce the computation burden [22].

Remark 6: The term $c_i\tilde{w}_i(t)$, $i = 1, 2, \ldots, N$, is involved to improve the robustness of the NN approximators to avoid the case when $\tilde{w}_i(t)$ shifts to very high values when the control gain Γ_i is large [20].

The main theoretical results of this study are summarised in the following theorem.

Theorem 1: Under Assumptions 1–5 and the conditions in Proposition 1, the leader-following consensus problem in Definition 1 can be solved by the neuro-adaptive control scheme (15) with NN adaptive law (31) and the distributed observer (7) for bounded initial condition $x_i(0)$ and $\tilde{w}_i(0)$. Moreover, the feedback control gain for (30) can be designed as

$$\beta_i(t) = \beta_{i0} + \beta_{i1}(t) \tag{32}$$

in which $\beta_{i0} \ge 2 + \chi_{\max}$ and

$$\beta_{i1}(t) = \frac{\gamma_i}{2}\left(1 + \frac{1}{(1-\tau_d) \parallel e_i \parallel^2}\int_{t-\tau_{\max}}^{t}\rho_i^2(x_i(s))\,\mathrm{d}s\right) \tag{33}$$

with design constants γ_i, $i = 1, 2, \ldots, N$.

Proof: For the case of $x_i \in \Omega_{r_i}^o$, consider the following Lyapunov candidate:

$$V(t) = V_e(t) + \frac{1}{2}\sum_{i=1}^{N}\Gamma_i^{-1}\hat{w}_i^2(t), \tag{34}$$

where $\hat{w}_i(t) = \tilde{w}_i(t) - w_i^*$.

Taking the time derivative of (34) and considering (25), one has

$$\begin{aligned}\dot{V}(t) \le \sum_{i=1}^{N}\Bigg((1+\chi_{\max}) \parallel e_i \parallel^2 + e_i^{\mathrm{T}}u_i^{et}(t) + e_i^{\mathrm{T}}f_i(x_i) \\ + \frac{\rho_i^2(x_i)}{2(1-\tau_d)}\Bigg) + \sum_{i=1}^{N}\Gamma_i^{-1}\hat{w}_i(t)\dot{\tilde{w}}_i(t) + \frac{Nd_M^2}{2}.\end{aligned} \tag{35}$$

Substituting (28) into (35) yields

$$\begin{aligned}\dot{V}(t) \le \sum_{i=1}^{N}\Bigg((1+\chi_{\max}) \parallel e_i \parallel^2 + e_i^{\mathrm{T}}u_i^{et}(t) + e_i^{\mathrm{T}}W_i^{*\mathrm{T}}S_i(x_i) \\ + e_i^{\mathrm{T}}\varepsilon_i(x_i) + \frac{\rho_i^2(x_i)}{2(1-\tau_d)}\Bigg) + \sum_{i=1}^{N}\Gamma_i^{-1}\hat{w}_i(t)\dot{\tilde{w}}_i(t) \\ + \frac{Nd_M^2}{2}.\end{aligned} \tag{36}$$

Applying the facts that

$$e_i^{\mathrm{T}}W_i^{*\mathrm{T}}S_i(x_i) \le \| e_i \|^2 \| W_i^{*\mathrm{T}}S_i(x_i) \|^2 + \frac{1}{4} \le w_i^* \| e_i \|^2 \| S_i(x_i) \|^2 + \frac{1}{4}, \tag{37}$$

$$e_i^{\mathrm{T}}\varepsilon_i(x_i) \le \| e_i \|^2 + \frac{1}{4} \| \varepsilon_i(x_i) \|^2 \le \| e_i \|^2 + \frac{1}{4}\varepsilon_M^2 \tag{38}$$

gives

$$\begin{aligned}\dot V(t) \le & \sum_{i=1}^{N}\Big((2+\chi_{\max}) \| e_i \|^2 + e_i^{\mathrm{T}}u_i^{et}(t) \\ & + w_i^* \| e_i \|^2 \| S_i(x_i) \|^2 + \frac{\rho_i^2(x_i)}{2(1-\tau_d)}\Big) \\ & + \sum_{i=1}^{N}\Gamma_i^{-1}\hat w_i(t)\dot{\hat w}_i(t) + \frac{N}{4} + \frac{N\varepsilon_M^2}{4} + \frac{Nd_M^2}{2}.\end{aligned} \tag{39}$$

Substituting (30) and (31) into (39) yields

$$\begin{aligned}\dot V(t) \le & \sum_{i=1}^{N}\Big((2+\chi_{\max}-\beta_i(t)) \| e_i \|^2 - \tilde w_i \| S_i(x_i) \|^2 \| e_i \|^2 \\ & - \frac{1}{2(1-\tau_d)}\rho_i^2(x_i) + w_i^* \| e_i \|^2 \| S_i(x_i) \|^2 \\ & + \frac{\rho_i^2(x_i)}{2(1-\tau_d)}\Big) + \sum_{i=1}^{N}\hat w_i(t)(\| S_i(x_i) \|^2 \| e_i \|^2 - c_i\tilde w_i(t)) \\ & + \frac{N}{4} + \frac{N\varepsilon_M^2}{4} + \frac{Nd_M^2}{2} \\ = & \sum_{i=1}^{N}((2+\chi_{\max}-\beta_i(t)) \| e_i \|^2) - \sum_{i=1}^{N}c_i\hat w_i(t)\tilde w_i(t) \\ & + \frac{N}{4} + \frac{N\varepsilon_M^2}{4} + \frac{Nd_M^2}{2}.\end{aligned} \tag{40}$$

Notice that $\hat w_i(t)\tilde w_i(t) = \hat w_i^2(t)/2 + \tilde w_i^2(t)/2 - w_i^{*2}(t)/2$, which implies $-c_i\hat w_i(t)\tilde w_i(t) \le -c_i((\hat w_i^2(t))/2) + c_i((w_i^{*2}(t))/2)$, then

$$\begin{aligned}\dot V(t) \le & \sum_{i=1}^{N}((2+\chi_{\max}-\beta_i(t)) \| e_i \|^2) - \frac{1}{2}\sum_{i=1}^{N}c_i\hat w_i^2(t) \\ & + \frac{1}{2}\sum_{i=1}^{N}c_iw_i^{*2}(t) + \frac{N}{4} + \frac{N\varepsilon_M^2}{4} + \frac{Nd_M^2}{2}.\end{aligned} \tag{41}$$

Denote $\theta_0 = 1/2\sum_{i=1}^{N}c_iw_i^{*2}(t) + N/4 + (N\varepsilon_M^2)/4 + (Nd_M^2)/2$. Obviously, θ_0 is a positive constant depending on design parameters. Following (32) and (33), one has

$$\begin{aligned}\dot V(t) \le & \sum_{i=1}^{N}\left(-\frac{\gamma_i}{2}\left(1 + \frac{1}{(1-\tau_d) \| e_i \|^2}\int_{t-\tau_{\max}}^{t}\rho_i^2(x_i(s))\,\mathrm{d}s\right) \| e_i \|^2 \right) \\ & - \frac{1}{2}\sum_{i=1}^{N}c_i\hat w_i^2(t) + \theta_0 \\ \le & -\frac{1}{2}\theta_1\sum_{i=1}^{N}\left(\| e_i \|^2 + \frac{1}{1-\tau_d}\int_{t-\tau_{\max}}^{t}\rho_i^2(x_i(s))\,\mathrm{d}s\right) \\ & - \frac{1}{2}\theta_1\sum_{i=1}^{N}\Gamma_i^{-1}c_i\hat w_i^2(t) + \theta_0 \\ \le & -\frac{1}{2}\theta_1\sum_{i=1}^{N}\left(\| e_i \|^2 + \frac{1}{1-\tau_d}\int_{t-\tau_i(t)}^{t}\rho_i^2(x_i(s))\,\mathrm{d}s\right) \\ & - \frac{1}{2}\theta_1\sum_{i=1}^{N}\Gamma_i^{-1}c_i\hat w_i^2(t) + \theta_0,\end{aligned} \tag{42}$$

where $\theta_1 = \min\{\gamma_1, \gamma_2, \ldots, \gamma_N, \Gamma_1c_1, \Gamma_2c_2, \ldots, \Gamma_Nc_N\}$.

Considering (34), the inequality (42) can be rewritten as

$$\dot V(t) \le -\theta_1V(t) + \theta_0. \tag{43}$$

Integrating both sides of (43) from time 0 to t yields

$$V(t) \le \frac{\theta_0}{\theta_1} + \left(V(0) - \frac{\theta_0}{\theta_1}\right)\exp(-\theta_1t). \tag{44}$$

Since $V(t) \ge 1/2\sum_{i=1}^{N}e_i(t)^{\mathrm{T}}e_i(t)$, we have

$$\| e_i \| \le \sqrt{2V(0)\exp(-\theta_1t) + \frac{2\theta_0}{\theta_1}(1-\exp(-\theta_1t))}. \tag{45}$$

For the case of $x_i \in \Omega_{r_i}$, because r_i in an arbitrarily small constant, the consensus tracking performance has been guaranteed.

Finally, combining with Proposition 1, we have

$$\lim_{t\to\infty} \| x_i(t) - y(t) \| \le \varpi, \quad \forall\ i = 1, 2, \ldots, N \tag{46}$$

where $\varpi = \sqrt{2\theta_0/\theta_1}$, which completes our proof. □

Remark 7: From the definition of ϖ, a better observer-based consensus tracking performance can be guaranteed by smaller θ_0 and larger θ_1, which can be obtained by increasing design constants γ_i, Γ_i, q_i and decreasing c_i appropriately.

Remark 8: Through the proof of Theorem 1, it is clear that the consensus errors of all followers are uniformly bounded for any bounded given initial condition $x_i(0) \in \Omega_{x_i(0)} \subset \mathbb{R}^m$. Also, it is worth noting that $\Omega_{x_i(0)}$ could be as large as possible in practice since it is not involved in the controller, which indicates that the analytic results in Theorem 1 are semi-global [32]. In other words, all followers will track the leader's output signals with residual errors being semi-globally uniformly ultimately bounded (SGUUB) [24]. Furthermore, the positive scalars ε_M, d_M are used for theoretical analysis only, and none of them is involved in the control protocol either, which means that they could be actually unknown.

The following corollary is natural and presented without proof.

Corollary 1: When the state delays of the follower agents are unknown but static constants bounded by $\tau_{\max}$, then the error feedback control term degenerated a bit into the following form:

$$u_i^{et}(t) = \begin{cases} -\beta_i(t)e_i - \tilde w_i(t) \| S_i(x_i) \|^2 e_i \\ -\frac{1}{2}e_i^{-1}\rho_i^2(x_i), & x_i \in \Omega_{r_i}^o, \\ 0, & x_i \in \Omega_{r_i} \end{cases} \tag{47}$$

with

$$\beta_{i1}(t) = \frac{\gamma_i}{2}\left(1 + \frac{1}{\| e_i \|^2}\int_{t-\tau_{\max}}^{t}\rho_i^2(x_i(s))\,\mathrm{d}s\right). \tag{48}$$

With all the other conditions in Theorem 1 hold, the consensus tracking problem in Definition 1 can also be solved.

Moreover, if all the time-delays of the followers are zeros as in [32], we can accordingly turn off all the delay rejection terms in the controllers and obtain the following corollary.

Corollary 2: Under Assumptions 1, 2 and 4 and Proposition 1, if the state delays of the follower agents $\tau_i(t)$ are known as 0, then the consensus tracking problem in Definition 1 can be solved by the controller (15) with observer (7), compensation term (16), feedback term

$$u_i^{et}(t) = -\beta_ie_i - \tilde w_i(t) \| S_i(x_i) \|^2 e_i \tag{49}$$

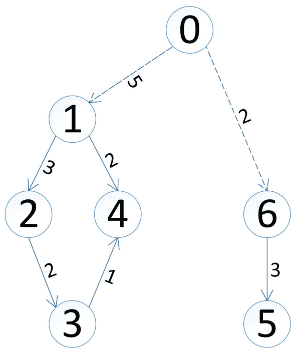

Fig. 1 *Communication topology among agents*

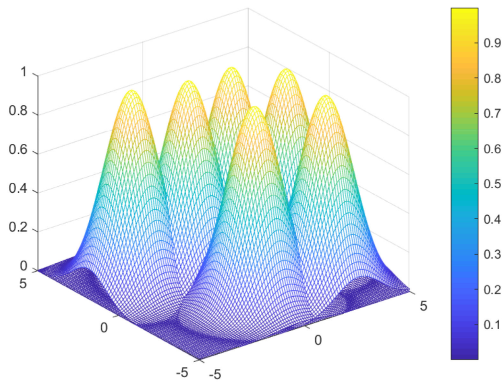

Fig. 2 *Response areas of hidden neurons of the RBFNNs*

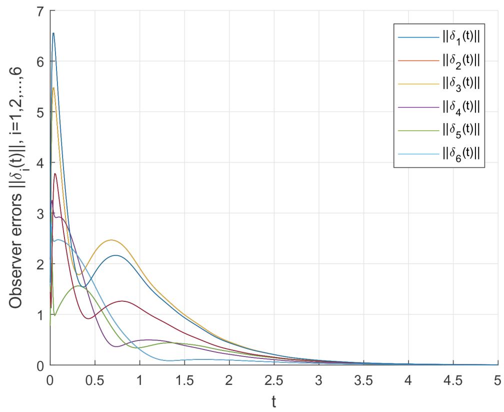

Fig. 3 *Observer errors* $\| \delta_i(t) \|$

and NN adaptive law (31) for bounded initial condition $x_i(0)$ and $\tilde{w}_i(0)$. Moreover, the feedback gain β_i is designed as

$$\beta_i \geq \frac{3+\gamma_i}{2} + \chi_{\max} \tag{50}$$

with design constants $\gamma_i, i = 1, 2, \ldots, N$.

Proof: The proof is similar to that of Theorem 1 when considering the Lyapunov function candidate

$$V(t) = V_1(t) + \frac{1}{2}\sum_{i=1}^{N} \Gamma_i^{-1} \hat{w}_i^2(t), \tag{51}$$

thus will be omitted due to the space limitation. □

4 Simulation examples

A numerical example is presented in this section to illustrate the effectiveness of the theoretical results. We only consider the situation with time-varying delays in MAS as it is more general.

Consider a MAS of six follower agents and a single leader. The communication network topology is depicted in Fig. 1 which satisfies Assumption 1.

For simplicity, we equip the agents with the same RBFNNs, which contain six hidden neurons with $v_i, i = 1, 2, \ldots, 6$, evenly distributed in the square area $[-3, 3] \times [-3, 3]$ and same widths $b_i = 1.5$. The response areas of the hidden neurons are visualised as in Fig. 2.

The leader agent is assumed as the two-mass-spring system with a single force input [39] whose dynamics can be modelled by (1) with

$$\begin{aligned} x_0(t) &= (x_{01}(t), x_{02}(t), x_{03}(t), x_{04}(t))^{\mathrm{T}}, \\ g(x_0(t), t) &= (0, 0.5\sin(x_{02}(t)) + 8\cos(8t), 0, 0)^{\mathrm{T}}, \\ \hat{A} &= \begin{pmatrix} 0 & 1 & 0 & 0 \\ \dfrac{-k_1 - k_2}{m_1} & 0 & \dfrac{k_2}{m_2} & 0 \\ 0 & 0 & 0 & 1 \\ \dfrac{k_2}{m_2} & 0 & \dfrac{-k_2}{m_2} & 0 \end{pmatrix}, \end{aligned} \tag{52}$$

where $m_1 = m_2 = 1.2$, spring constants $k_1 = 1.0$ and $k_2 = 1.5$. The output matrix is chosen as $\hat{C} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ and it can be easily checked that $(\hat{C}, \hat{A})$ is detectable. The dynamics of each follower can be described by (3) with matrix $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, nonlinear function

$$\begin{aligned} f_i(x_i(t)) &= (x_{i1}(t)\sin(i + x_{i1}(t)), 2\cos(x_{i2}(t)))^{\mathrm{T}}, \\ h_i(x_i(t)) &= (x_{i1}(t)\cos(x_{i1}(t)), x_{i2}(t)\cos(i + x_{i2}(t)))^{\mathrm{T}}, \end{aligned} \tag{53}$$

external disturbances $d_i(t) = (d_{i1}(t), d_{i2}(t))^{\mathrm{T}}, i = 1, 2, \ldots, 6$, and $d_{ij}(t)$ are chosen as random but bounded by $|d_{ij}(t)| \leq 0.5$, $j = 1, 2$. Time varying delays for each agent are $\tau_1 = 0.4(e^t/(1+e^t))$, $\tau_2 = 0.5(e^t/(1+e^t))$, $\tau_3 = 0.6(e^t/(1+e^t))$, $\tau_4 = 0.7|\sin(0.5t)|$, $\tau_5 = 0.8|\sin(0.5t)|$, $\tau_6 = 0.9|\sin(0.5t)|$, respectively. Obviously, by choosing $\psi = 0.5$, $\rho_i(x_i(t)) = \sqrt{x_{i1}^2(t) + x_{i2}^2(t)}, i = 1, 2, \ldots, 6$, $d_M = 0.5$, $\tau_d = 0.5$, and $\tau_{\max} = 1$, the Assumptions 1–5 hold. The initial state of the leader is $x_0(0) = (0, 0, 0, 0)^{\mathrm{T}}$, and the initial positions of the followers are randomly chosen from normal distribution with zero mean and standard derivation 2. Moreover, initial observers are chosen randomly from standard normal distribution, and the NN weights are initialised as zero matrices.

Following (6), we get $\underline{\lambda} = 2.3150$. Set $\alpha = 15$, $c_0 = 0.2$ and solve LMI (12), we then have

$$F = -\begin{pmatrix} 1.8994 & 1.6970 & 0.2204 & 0.4784 \\ 11.1735 & 10.2524 & 0.4784 & 2.8884 \end{pmatrix}^{\mathrm{T}}.$$

The adaptive NN controllers are given in (15), (16), (30) with feedback gain $\beta_i(t)$ given in (32), (33), where $\beta_{i0} = 210$, $\gamma_i = 0.6$, $r_i = 10^{-5}$. The updating law of NN follows (31) with $\Gamma_i = 100$, $c_i = 0.02$. We use $e_i^c(t) = (e_{i1}^c(t), e_{i2}^c(t))^{\mathrm{T}} = x_i(t) - y(t)$ to denote the tracking consensus error vector of follower i, and $u_i(t) = (u_{i1}(t), u_{i2}(t))^{\mathrm{T}}, i = 1, 2, \ldots, 6$.

Fig. 3 shows the evolutionary of observer states. Figs. 4 and 5 help to visualise consensus tracking performances. Fig. 6 and 7 show the control efforts. Finally the NN adaptive parameters are visualised in Fig. 8.

To verify the disturbance rejection properties of the designed controllers, in Fig. 9 are plotted the profiles of the overall tracking errors $E(t) = \sqrt{1/6\sum \| x_i(t) - y(t) \|^2}$ with and without the above randomly bounded external disturbances.

To further verify the time-delay rejection properties of the designed controllers, we assume the time delays $\tau_i(t) = 0$ as in

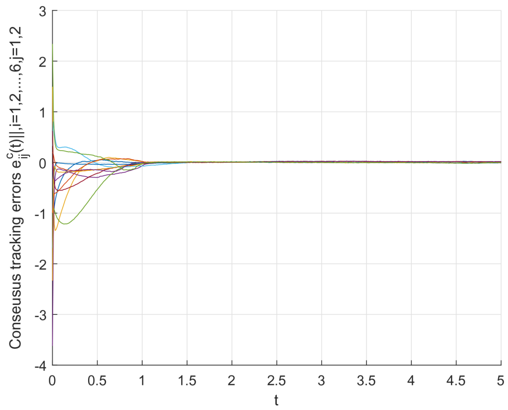

Fig. 4 *Consensus tracking errors $e^c_{ij}(t)$*

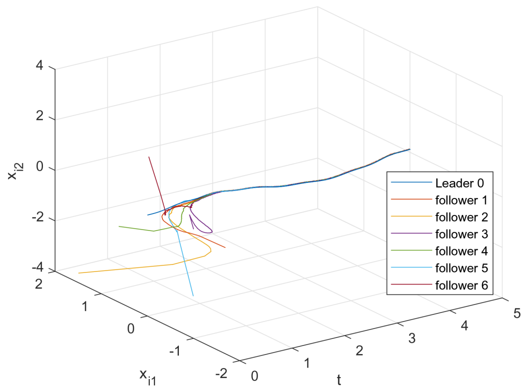

Fig. 5 *Trajectories of all agents*

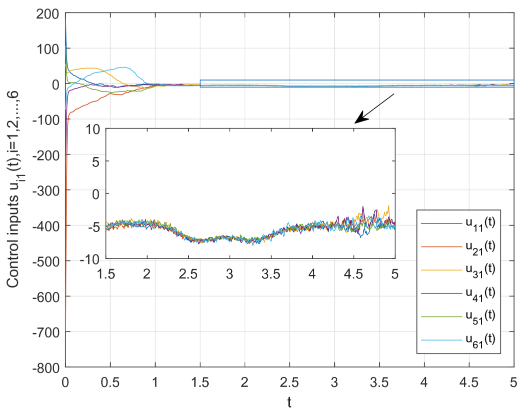

Fig. 6 *Control input components $u_{i1}(t)$*

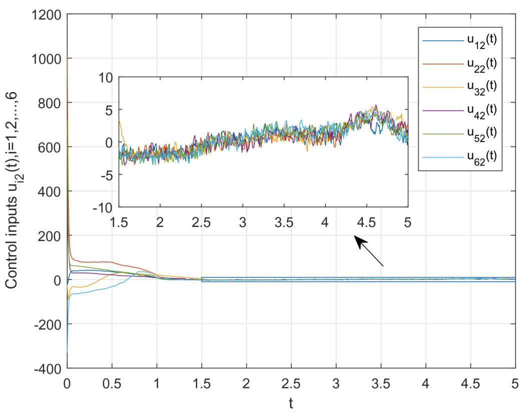

Fig. 7 *Control input components $u_{i2}(t)$*

Corollary 2. With the same design parameters c_0, α, Γ_i, c_i, γ_i above and $\beta_i = 210$, the consensus tracking problem can also be solved.

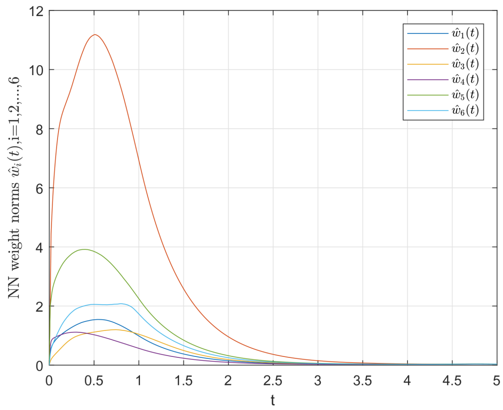

Fig. 8 *NN weight norms $\hat{w}_i(t)$*

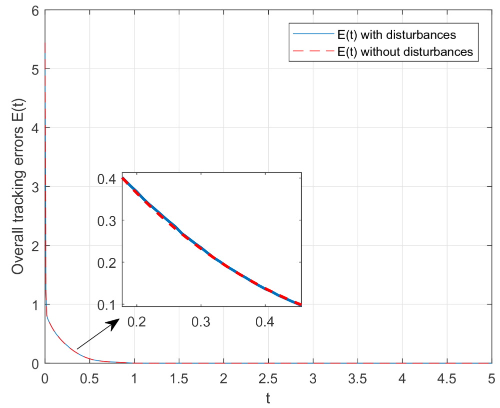

Fig. 9 *$E(t)$ with and without disturbances*

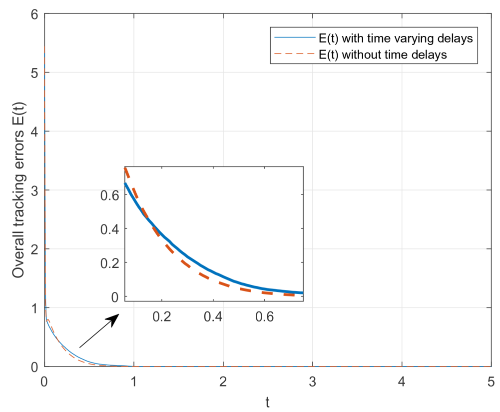

Fig. 10 *$E(t)$ with and without time varying delays*

Finally, Fig. 10 gives the profiles of $E(t)$ with and without the above time varying delays.

Remark 9: If some previous NN adaptive methods are used for this problem, for instance, the method of [20, 32], the size of weight matrices will be 6×2, which means that 12 parameters need to be updated online by each agent for the approximation task. However, in this study, only a scalar parameter is iterated for each agent so as to approximate its unknown nonlinear dynamical function, which is very efficient from the economic point of view.

5 Conclusion

An adaptive leader-following consensus strategy is developed for a class of nonlinear MAS with uncertainties and heterogeneities. In the concerned model, the dynamics and dimensions of the leader are different from those of the followers. Meanwhile, the dynamics of the followers involves unmodelled dynamics, unknown time-varying delays, and external disturbances. To complete the control task, a distributed full-order observer is designed for each follower to reconstruct the leader's state based on local information, then the uncertain nonlinearity and the time delays of agent's dynamics are compensated by employing RBFNNs and by choosing a well-designed Lyapunov–Krasovskii functional, respectively. Moreover, by appropriately cutting down the NN parameters to be updated, the computational burden is greatly alleviated compared with previous works. Finally, stability analysis is performed based on the Lyapunov stability theory and simulation results further verified the validity and efficiency of theoretical results. For future works, the authors are interested in the agreement under event-based schemes and the presence of communication delays.

6 References

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