Adaptive Exponential Consensus With Cooperative Exponential Parameter Identification Over Leaderless Directed Graphs

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***Abstract*—Due to the complex entanglement between distributed control and distributed estimation, adaptive multiagent dynamics over leaderless directed graphs are yet not completely understood. This happens even when the adaptive dynamics are based on established tools like model reference adaptive control (MRAC). This work starts from the observation that existing MRAC-based leaderless designs stop at asymptotic consensus, lacking of any guarantee for exponential consensus or parameter identification. The main contribution of this work is to show a new design departing from the existing ones in terms of exploiting persistence of excitation (PE): the proposed design is the first one attaining exponential consensus with exponential parameter identification over leaderless directed graphs. In the special case that the unknown parameters are homogeneous, PE can be relaxed to a weaker cooperative PE (C-PE) condition. The design is illustrated and verified alongside the state-of-the-art.**

***Index Terms*—Adaptive control, directed graphs, leaderless consensus, parameter identification, persistence of excitation (PE).**

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I. Introduction

IN THE past few decades, cooperative control of multiagent systems has arose multidisciplinary interests spanning connected vehicles [1], optimization and game theory [2], [3], microgrids [4], biological science [5], and so forth.

Since uncertainties are inevitable in most applications of multiagent systems, cooperative control of uncertain multiagent systems has been receiving increasing attention. To deal with uncertainty, different adaptive control techniques have been successfully applied to multiagent systems. For instance, neural networks and fuzzy logic systems have been used to tackle unknown nonlinear dynamics in multiagent systems [6], [7], [8], [9], [10], adaptive control with Nussbaum functions has been used to tackle unknown control directions in multiagent systems [11], and adaptive couplings have been used to tackle the unknown algebraic connectivity in multiagent systems [12], [13]. The celebrated model reference adaptive control (MRAC) has been recently applied to multiagent systems with parametric uncertainties [14], [15], [16]. In particular, model reference adaptive consensus (MRACon) has been proposed in [16], which provides a solution for leaderless consensus of multiagent systems with parametric uncertainties, and is effective when the communications graph among the agents is directed. Note that MRACon only leads to asymptotic consensus, lacking of any guarantee for exponential consensus or parameter identification.

It is known ever since [17] that adaptive control does not necessarily require perfect identification of the uncertainties, i.e., the parameter estimates may not converge to their true values. However, it is also widely accepted that convergence to the true parameters would result in a robust behavior [18]: this gives room to various parameter identification tools putting more focus on the conditions that allow to identify the uncertainties. Such conditions traditionally evolve around the celebrated persistence of excitation (PE), a condition allowing for exponential convergence to the true parameters [18, Corollary 4.3.1]. By resorting to consensus theory, the PE condition in multiagent systems can be relaxed to a weaker condition named cooperative PE (C-PE), provided the unknown parameters are homogeneous among all agents [19], [20], [21], [22]. Such scenarios include multiple robots with homogeneous unknown dynamics or acting in the same environment with unknown but homogeneous physical parameters such as gravity and temperature, among others. The PE condition has also been relaxed to interval excitation (IE) [23], [24], [25], [26], and, likewise, C-PE has been relaxed to C-IE [27]. For instance, [23] achieves IE via integrator filters: similar integrator filters have been used to derive the C-IE condition in multiagent systems [27]. However, as pointed out by [28], such integrator filters create monotonically increasing signals that may harm stability. Another technique that achieves IE is the concurrent learning method [29], which is at the price of a history stack to store past state information.

Apart from the interesting but still unsettled issue of relaxing PE and C-PE to IE and C-IE, in this work we focus on PE and C-PE, because these conditions still present some crucial open problems for multiagent systems. In fact, despite the advances in cooperative control and parameter identification of uncertain multiagent systems, several challenging problems arise due to the entanglement between the parameter estimates and the distributed controllers. Notably, previous works [19], [22] are applicable to decentralized control where the purpose is to make each agent track a local reference, i.e., a truly cooperative control goal is missing. Meanwhile, existing approaches for addressing parameter identification in leader-follower tracking [30], [31], containment control [32], and cooperative output regulation [33], all rely on a leader-follower setting with undirected communication graph among the followers. For multiagent systems without any leader or with directed communication graphs, cooperative control with parameter identification remain open problems. As pointed out by [16] and [34], cooperative control of uncertain leaderless agents over directed graphs is generally more challenging due to the lack of a global reference and the asymmetric communications.

Motivated by the above discussions, this article studies leaderless consensus with parameter identification of multiagent systems over directed graphs. In particular, we study exponential stability of the distributed adaptive closed loop since exponentially stable systems are known to possess a certain degree of tolerance to perturbations [35]. We consider agents with possibly heterogeneous parametric uncertainties and derive the corresponding results in the special case of homogeneous parametric uncertainties. The main contributions are summarized as follows.

1) When the uncertainties are heterogeneous and PE holds, we propose an improved MRACon achieving not only exponential consensus but also exponential parameter identification for the multiagent systems. We remark that the existing MRACon [16] only achieves asymptotic (not exponential) consensus.
2) When the uncertainties are homogeneous, the identification task can be accomplished in a cooperative way. This allows to modify the adaptation law of MRACon so as to exploit a C-PE condition, weaker than PE, but that still allows exponential consensus and exponential parameter identification.
3) To our best knowledge, the proposed design is the first framework for cooperative control with parameter identification over leaderless directed graphs. In fact, [19], [22] consider a decentralized setting rather than a cooperative setting, and [30], [31], [32], [33] consider leader-follower settings over undirected graphs.

The rest of this article is organized as follows. In Section II, we introduce some preliminaries, formulate the problem, and recall the baseline MRACon. In Sections III and IV, we present the main results which are the construction of two improved MRACon and their exponential convergence results. Simulation examples are provided in Section V, followed by conclusions in Section VI.

Notations: Let $\mathbb{R}$ and $\mathbb{R}_+$ denote the field of real numbers and positive real numbers, respectively. Denote with I and $\mathbf{1}$ the identity matrix and the column vector with all ones, respectively. Zero vectors and zero matrices are all denoted as 0 for brevity. The 2-norm (resp. induced 2-norm) of a vector (resp. a matrix) is denoted as $\| \bullet \|$. For a vector $a \in \mathbb{R}^n$, span(a) is the space spanned by vector a; we write $a \in \mathbb{L}_\infty$ if $\|a\|_\infty \triangleq \sup_{t\geq 0} \|a(t)\|$ exists. For a square matrix $A \in \mathbb{R}^{n\times n}$, let null($A$) denote its zero space, tr(A) its trace, and $\lambda(A)$ its eigenvalues. The matrix inequality $A \geq B$ means that $A - B$ is positive semidefinite. The real transpose and the complex conjugate transpose are denoted as $\bullet^\mathsf{T}$ and $\bullet^\mathsf{H}$, respectively. The real part of a complex number is denoted as $\Re(\bullet)$. For a group of matrices $a_1, \ldots, a_n$ of compatible dimensions, $\text{col}(a_1, \ldots, a_n) \triangleq (a_1^\mathsf{T}, \ldots, a_n^\mathsf{T})^\mathsf{T}$, and $\text{diag}(a_1, \ldots, a_n)$ is the block diagonal matrix consisting of $a_1, \ldots, a_n$ in order. Denote the index set $\mathcal{I}_N \triangleq \{1, 2, \ldots, N\}$.

II. Preliminaries

A. Graph Theory

A *directed graph* (or simply *digraph*) $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ is described by the node set $\mathcal{V} = \mathcal{I}_N$ and the edge set $\mathcal{E} = \{\mathcal{E}_{ij}, i \neq j | i \to j\}$ representing the interconnections between nodes. The weighted adjacency matrix $\mathcal{A} = (a_{ij}) \in \mathbb{R}^{N\times N}$ has non-negative entries with $a_{ij} > 0$ if $\mathcal{E}_{ji} \in \mathcal{E}$: in this case, node j is called an in-neighbor of node i. The entries of the Laplacian matrix $\mathcal{L} = (l_{ij}) \in \mathbb{R}^{N\times N}$ associated with $\mathcal{G}$ are defined as $l_{ij} = -a_{ij}$ for $i \neq j$, and $l_{ii} = \sum_{j=1}^N a_{ij}$. A digraph $\mathcal{G}$ is *weight-balanced* if $\sum_{j=1}^N a_{ij} = \sum_{j=1}^N a_{ji}$ for any $i \in \mathcal{V}$. A digraph $\mathcal{G}$ is *strongly connected* if there exists a directed path between any pair of nodes. A directed spanning tree (DST) $\bar{\mathcal{G}}(\mathcal{V}, \bar{\mathcal{E}}, \bar{\mathcal{A}})$ of $\mathcal{G}$ is a subgraph which contains a root (a node without in-neighbors), such that one can find a unique directed path from the root to every other node. In $\bar{\mathcal{G}}$, without loss of generality, let node 1 be the root and denote i_k as the unique in-neighbor of node $k+1$, $k \in \mathcal{I}_{N-1}$.

Suppose that $\mathcal{G}$ contains a DST $\bar{\mathcal{G}}$ and let us construct two matrices based on $\bar{\mathcal{G}}$. Construct $\Xi \in \mathbb{R}^{(N-1)\times N}$ and $Q \in \mathbb{R}^{(N-1)\times(N-1)}$ as

$$\Xi_{kj} = \begin{cases} -1, & \text{if } j = k+1 \\ 1, & \text{if } j = i_k \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

$$Q_{kj} = \sum_{c \in \bar{\mathcal{V}}_{j+1}} \left(\mathcal{L}_{k+1,c} - \mathcal{L}_{i_k,c} \right) \tag{2}$$

where $\bar{\mathcal{V}}_{j+1}$ represents the node set of the subtree of $\bar{\mathcal{G}}$ rooting at node $j+1$. Then, the following results hold.

Lemma 1 [2], [13], [36]: Let $\mathcal{G}$ contain a DST $\bar{\mathcal{G}}$. Then,

1) $0 = \lambda_1(\mathcal{L}) < \Re(\lambda_2(\mathcal{L})) \leq \ldots \leq \Re(\lambda_N(\mathcal{L}))$.
2) $\mathrm{null}(\mathcal{L}) = \mathrm{null}(\Xi) = \mathrm{span}(\mathbf{1}_N)$.
3) $\Xi\mathcal{L} = Q\Xi$.
4) $\lambda_i(Q) = \lambda_{i+1}(\mathcal{L})$, $i = 1, \ldots, N-1$.

Lemma 2 [37]: Let $\mathcal{G}$ be strongly connected. Then,

1) there exists a positive left eigenvector $r = (r_1, \ldots, r_N)^\mathsf{T} \in \mathbb{R}_+^N$ of $\mathcal{L}$ associated with the zero eigenvalue.
2) the matrix $\hat{\mathcal{L}} \triangleq R\mathcal{L} + \mathcal{L}^\mathsf{T}R$, where $R = \mathrm{diag}(r_1, \ldots, r_N)$, is a Laplacian matrix associated with an undirected connected graph.
3) $r = r_0\mathbf{1}_N$ with $r_0 \in \mathbb{R}_+$ if and only if $\mathcal{G}$ is weight-balanced.

B. PE/C-PE Signals and Minimum-Phase Filters

Definition 1 ([38], PE): A bounded signal $\phi(t) : [0, \infty) \to \mathbb{R}^n$ is persistently exciting, denoted by $\phi \in \mathrm{PE}(T, \epsilon)$, if there exist $T, \epsilon \in \mathbb{R}_+$ such that

$$\int_t^{t+T} \phi(\tau)\phi(\tau)^\mathsf{T} \mathrm{d}\tau \geq \epsilon I_n \qquad \forall t \geq 0.$$

Definition 2 ([19], [20], C-PE): A group of bounded signals $\phi_i(t) : [0, \infty) \to \mathbb{R}^n$ for $i \in \mathcal{I}_N$ is cooperatively PE, denoted by $\{\phi_i | i \in \mathcal{I}_N\} \in \text{C-PE}(T, \epsilon)$, if there exist $T, \epsilon \in \mathbb{R}_+$ such that

$$\int_t^{t+T} \sum_{i=1}^N \phi_i(\tau)\phi_i(\tau)^\mathsf{T} \mathrm{d}\tau \geq \epsilon I_n \qquad \forall t \geq 0.$$

Definition 3 [18], [38]: For an LTI system $y = G(s)u$, the filter $G(s)$ is

1) *proper* if $G(\infty)$ is finite;
2) *minimum-phase* if its numerator polynomial is Hurwitz;
3) *stable* if its denominator polynomial is Hurwitz.

The following lemma is recalled from [18, Lemma 4.8.3].

Lemma 3 [18]: If a bounded signal $\phi : [0, \infty) \to \mathbb{R}^n$ is PE and $\dot{\phi} \in \mathbb{L}_\infty$, and a filter $G(s)$ is proper, stable and minimum-phase, then $G(s)\phi$ is bounded and also PE.

A natural question is whether Lemma 3 can be extended to the C-PE case. The following result gives a positive answer, which also avoids making indirect assumptions on the filtered regressors, as is indeed done in [31], [32], and [33].

Lemma 4: If a group of bounded signals $\phi_i : [0, \infty) \to \mathbb{R}^n$ is C-PE and $\dot{\phi}_i \in \mathbb{L}_\infty$ for $i \in \mathcal{I}_N$, and a filter $G(s)$ is proper, stable and minimum-phase, then each $G(s)\phi_i$ is bounded and the group of $G(s)\phi_i$ is C-PE.

Proof: See Appendix A. ■

C. Problem Statement and Baseline MRACon

Consider N agents with parametric uncertainties interacting over a digraph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$. The dynamics of agent i, $i \in \mathcal{V}$, is

$$\dot{x}_i(t) = Ax_i(t) + B(f_i(x_i(t), t) + u_i(t)) \tag{3}$$

where $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^m$ are the state and input vector, respectively, and the pair $(A; B)$ is stabilizable, known, and with B full column rank. Parametric uncertainty arises from the following standard assumption on f_i [16], [28].

Assumption 1: For each agent $i \in \mathcal{V}$, the function f_i can be linearly parameterized, i.e.,

$$f_i(x_i(t), t) = W_i^\mathsf{T}\phi_i(x_i(t), t) \tag{4}$$

where $W_i \in \mathbb{R}^{q\times m}$ is an *unknown* constant matrix, and $\phi_i : \mathbb{R}^n \times [0, \infty) \to \mathbb{R}^q$, often called the regressor, is known and such that $\phi_i, \dot{\phi}_i \in \mathbb{L}_\infty$.

Under suitable conditions imposed on the digraph $\mathcal{G}$ and on the regressors ϕ_i, the problem of interest is to design u_i and an estimate $\hat{W}_i(t)$ of W_i so as to achieve exponential consensus with exponential parameter identification defined as follows.

Definition 4: The agents (3) are said to achieve exponential consensus and exponential parameter identification, respectively, if

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0 \qquad \forall i, j \in \mathcal{V}$$
$$\lim_{t\to\infty} \|\tilde{W}_i(t)\| = 0 \qquad \forall i \in \mathcal{V}$$

with $\tilde{W}_i(t) \triangleq \hat{W}_i(t) - W_i$, where the convergence is exponential after $t \geq T^*$ for some $T^* \in \mathbb{R}_+$. Specifically, there exist ρ, $\eta > 0$ such that $\lim_{t\to\infty} \|x_i(t) - x_j(t)\| \leq \rho \mathrm{e}^{-\eta(t-T^*)}\|x_i(T^*) - x_j(T^*)\|$ and $\lim_{t\to\infty} \|\tilde{W}_i(t)\| \leq \rho \mathrm{e}^{-\eta(t-T^*)}\|\tilde{W}_i(T^*)\|$ for all $t \geq T^*$: in this case, the convergence is exponential at the rate of (no less than) η.

Remark 1: The delay T^* is typically introduced in related literature [31], [32], [33], [39] to account for the initial transient of the adaptive closed-loop.

Let us now recall a baseline design over leaderless directed graphs (see Fig. 1). This design was originally reported in [16] and called MRACon (here and in the sequel the arguments of the functions are omitted when clear from the context)

$$\begin{aligned} u_i &= Ke_i - \hat{W}_i^\mathsf{T}\phi_i + \gamma_i \\ e_i &= x_i - z_i \\ \dot{z}_i &= Az_i + B\gamma_i \\ \gamma_i &= \alpha K \sum_{j=1}^N a_{ij}\big(x_i - x_j\big) \end{aligned} \tag{5}$$

where z_i is the state of a virtual local reference model constructed for agent i, $\alpha \in \mathbb{R}_+$ and $K \in \mathbb{R}^{m\times n}$ are constant gains, and $\hat{W}_i$ is updated via

$$\dot{\hat{W}}_i = \Gamma\phi_i e_i^\mathsf{T}PB. \tag{6}$$

Here, $\Gamma \in \mathbb{R}^{q\times q}$ and $P \in \mathbb{R}^{n\times n}$ are constant and positive definite. The following result is recalled from [16].

Proposition 1 [16]: Suppose that the digraph $\mathcal{G}$ contains a DST. Let $\alpha \geq [1/2\Re(\lambda_2(\mathcal{L}))]$ and $K = -B^\mathsf{T}P$ with P being the unique solution of the algebraic Riccati equation $A^\mathsf{T}P + PA - PBB^\mathsf{T}P + I_n = 0$. Then, the agents (3) will achieve asymptotic consensus under (5)-(6).

The above Proposition 1 provides asymptotic (but not exponential) consensus. Most notably, parameter convergence of $\hat{W}_i$ to W_i is left unaddressed. In the following section, two novel classes of MRACon will be proposed, both guaranteeing exponential consensus and exponential parameter identification. To close the section, a lemma on exponential stability under decaying disturbances is introduced.

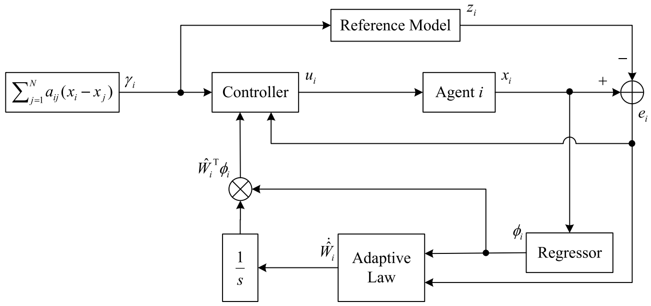

Fig. 1. Block diagram of the baseline MRACon framework proposed in [16]. The parameter identification is open-loop.

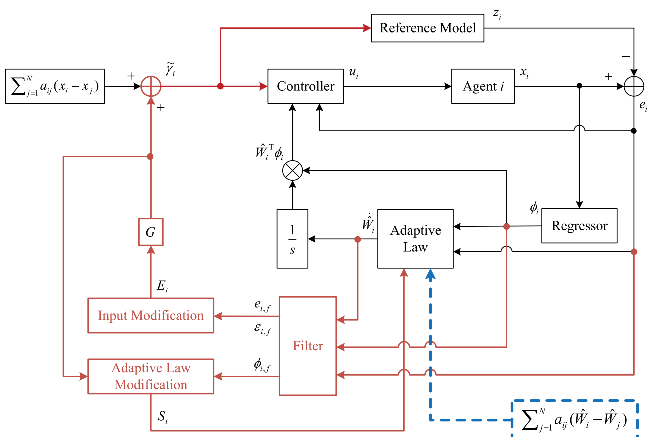

Fig. 2. Block diagram of the proposed improved MRACon framework. The red part closes the loop of parameter identification and will be studied in Section III. The blue part is for cooperative parameter identification and will be studied in Section IV.

Lemma 5 [40]: Consider the following system:

$$\dot{x} = Fx + F_1(t)x + F_2(t)$$

where $x \in \mathbb{R}^n$, $F \in \mathbb{R}^{n\times n}$ is Hurwitz, $F_1(t) \in \mathbb{R}^{n\times n}$ and $F_2(t) \in \mathbb{R}^n$ are bounded and continuous for all $t \geq t_0$. Then, as $t \to \infty$,

1) if $F_1(t), F_2(t) \to 0$ (exponentially), then $x(t) \to 0$ (exponentially);
2) if $F_1(t) = 0$, $F_2(t)$ decays exponentially at the rate of α, and $\alpha < \beta$ where $\beta = \min_k\{\Re(\lambda_k(-F))\}$, then $x(t) \to 0$ exponentially at the rate of α.

III. Adaptive Exponential Consensus With Exponential Parameter Identification

The main idea to achieve parameter identification is to close the loop of the parameter estimate (6) for each agent, i.e., to involve the parameter estimation error for adaptation (see Fig. 2). The idea is inspired by [28], [41], and [42], which address identification and control problems of a single uncertain plant. To accommodate for the multiagent setting, a standard connectivity assumption common in consensus control (see [16], [36], etc.) is introduced as follows.

Assumption 2: The digraph $\mathcal{G}$ contains a DST $\bar{\mathcal{G}}$.

Assumption 3: The regressor ϕ_i in (4) $\forall i \in \mathcal{V}$, is PE: $\phi_i \in \mathrm{PE}(T_i, \epsilon_i)$ for some $T_i, \epsilon_i \in \mathbb{R}_+$.

Consider the following proposed improved MRACon law for each agent $i \in \mathcal{V}$ as:

$$\begin{aligned} u_i &= Ke_i - \hat{W}_i^{\mathsf{T}}\phi_i + \tilde{\gamma}_i \\ e_i &= x_i - z_i \\ \dot{z}_i &= Az_i + B\tilde{\gamma}_i \\ \tilde{\gamma}_i &= \alpha K \sum_{j=1}^{N} a_{ij}\big(x_i - x_j\big) + GE_i \end{aligned} \tag{7}$$

where the update law of $\hat{W}_i$ will be specified later, $G = (B^{\mathsf{T}}B)^{-1}B^{\mathsf{T}}$, and

$$E_i = (A + BK)e_{i,f} + B\varepsilon_{i,f} - \frac{e_i - e_{i,f}}{k} \tag{8}$$

with $k \in \mathbb{R}_+$ and local filtered variables $e_{i,f}, \varepsilon_{i,f}$ calculated as

$$\begin{aligned} k\dot{e}_{i,f} + e_{i,f} &= e_i \\ k\dot{\varepsilon}_{i,f} + \varepsilon_{i,f} &= k\dot{\hat{W}}_i^{\mathsf{T}}\phi_{i,f} \\ k\dot{\phi}_{i,f} + \phi_{i,f} &= \phi_i. \end{aligned} \tag{9}$$

These variables $e_{i,f}, \varepsilon_{i,f}, \phi_{i,f}$ can be easily obtained by applying the filter $H(s) \triangleq (1/[ks+1])$ to $e_i, k\dot{\hat{W}}_i^{\mathsf{T}}\phi_{i,f}, \phi_i$, respectively.

In order to construct the update law, auxiliary matrix updates are needed, that are introduced via the following result:

Proposition 2: Consider the matrices updated via

$$\begin{aligned} \dot{M}_i &= -lM_i + \delta\phi_{i,f}\phi_{i,f}^{\mathsf{T}}, \qquad M_i(0) = 0 \\ \dot{S}_i &= -lS_i - \delta\phi_{i,f}E_i^{\mathsf{T}}G^{\mathsf{T}} - M_i\dot{\hat{W}}_i \qquad S_i(0) = 0 \end{aligned} \tag{10}$$

where $l, \delta \in \mathbb{R}_+$ are design gains with l often referred to as the forgetting factor. Then,

1) $S_i(t) = -M_i(t)\tilde{W}_i(t)\quad \forall t \geq 0\quad \forall i \in \mathcal{V}$.
2) under Assumption 3, there exists a $\sigma \in \mathbb{R}_+$, such that $M_i(t) \geq \sigma I_q\quad \forall t \geq T_i,\ i \in \mathcal{V}$.

Proof: See Appendix B. ■

We are now in a position to design the update law $\dot{\hat{W}}_i$ as

$$\dot{\hat{W}}_i = \Gamma\Big(\phi_i e_i^{\mathsf{T}}PB + \kappa_1 S_i\Big) \tag{11}$$

where $\kappa_1 \in \mathbb{R}_+$. The main difference between the baseline (5) and the proposed (7) is the following: the latter contains the additional term E_i that depends on the update law (11). In other words, an additional feedback from the parameter update law is created, which requires a dedicated stability analysis.

The following theorem summarizes the main result of this section.

Theorem 1: Let Assumptions 1–3 hold, and choose $\alpha \geq (1/[2\Re(\lambda_2(\mathcal{L}))])$ and $K = -B^{\mathsf{T}}P$ as in Proposition 1. Then, with the proposed improved MRACon (7)–(11), it holds that

1) the agents (3) achieve exponential consensus with exponential parameter identification, where T^* in Definition 4 can be specified as $T^* = \max_i\{T_i\}$;
2) if $m = n$, let α further satisfy

$$\alpha > \frac{1}{2\Re(\lambda_2(\mathcal{L}))} + \frac{\varrho - 2}{4\Re(\lambda_2(\mathcal{L}))\lambda_{\min}\big(PBB^{\mathsf{T}}P\big)} \tag{12}$$

where $\varrho \triangleq \min\{([1 + \lambda_{\min}(PBB^{\mathsf{T}}P)]/\lambda_{\max}(P)), 2\kappa_1\sigma\lambda_{\min}(\Gamma)\} > 0$, then, the exponential convergence in both consensus and parameter identification is at the rate of $(\varrho/2)$.

Proof: The proof of statement 1 will be divided into three parts, i.e., exponential convergence of e_i and $\tilde{W}_i$, exponential consensus of z_i, and exponential consensus of x_i.

A. Exponential Convergence of e_i and $\tilde{W}_i$

The dynamics of the error systems e_i, $\tilde{W}_i$ can be obtained as

$$\begin{aligned}\dot{e}_i &= (A+BK)e_i - B\tilde{W}_i^{\mathsf{T}}\phi_i \\ \dot{\tilde{W}}_i &= \Gamma\left(\phi_i e_i^{\mathsf{T}} PB - \kappa_1 M_i \tilde{W}_i\right)\end{aligned} \tag{13}$$

where we have used Proposition 2.

Consider, for each agent $i \in \mathcal{V}$, the candidate Lyapunov function

$$V_i\big(e_i, \tilde{W}_i\big) = e_i^{\mathsf{T}} P e_i + \operatorname{tr}\left(\tilde{W}_i^{\mathsf{T}}\Gamma^{-1}\tilde{W}_i\right). \tag{14}$$

Analyzing the time derivative of V_i along the trajectories of (13) yields

$$\begin{aligned}\dot{V}_i &= e_i^{\mathsf{T}}\left(PA + A^{\mathsf{T}}P - 2PBB^{\mathsf{T}}P\right)e_i - 2e_i^{T}PB\tilde{W}_i^{\mathsf{T}}\phi_i \\ &\quad + 2\operatorname{tr}\left(\tilde{W}_i^{T}\left(\phi_i e_i^{\mathsf{T}}PB - \kappa_1 M_i\tilde{W}_i\right)\right) \\ &= -e_i^{\mathsf{T}}\left(I_n + PBB^{\mathsf{T}}P\right)e_i - 2\kappa_1 \operatorname{tr}\left(\tilde{W}_i^{\mathsf{T}} M_i \tilde{W}_i\right)\end{aligned} \tag{15}$$

where the definition of P and the fact that $\operatorname{tr}(XY) = \operatorname{tr}(YX)$ for any matrices X and Y of compatible dimension has been used to obtain the last equality.

By Proposition 2, $M_i(t) > \sigma I_q \quad \forall t \geq T_i$. Then, it follows from (15) and the definition of ϱ that

$$\dot{V}_i(t) \leq -\varrho V_i(t) \qquad \forall t \geq T_i. \tag{16}$$

As such, the Lyapunov direct theorem allows to conclude that e_i and $\tilde{W}_i$ converge exponentially to zero after $t \geq T_i$.

B. Exponential Consensus of z_i

Let us denote $z = \operatorname{col}(z_1, \ldots, z_N)$, and define x, e, E in a similar way. From (7), the dynamics of z can be obtained as

$$\begin{aligned}\dot{z} &= (I_N \otimes A)z + \alpha(\mathcal{L}\otimes BK)x + (I_N \otimes BG)E \\ &= (I_N \otimes A + \alpha\mathcal{L}\otimes BK)z + \alpha(\mathcal{L}\otimes BK)e + (I_N\otimes BG)E.\end{aligned}$$

Under Assumption 2, define Ξ and Q as in (1)-(2) and let $\bar{z} = (\Xi \otimes I_n)z$. Then, based on Lemma 1, one has

$$\begin{aligned}\dot{\bar{z}} &= (I_{N-1}\otimes A + \alpha Q \otimes BK)\bar{z} \\ &\quad + \alpha(\Xi\mathcal{L}\otimes BK)e + (\Xi \otimes BG)E.\end{aligned} \tag{17}$$

Moreover, $\bar{z} = 0$ if and only if z_i reach consensus. Note that e and E converge exponentially to zero after $t \geq \max_i\{T_i\}$. In particular, E_i decay exponentially after $t \geq T_i$ since $E_i = B\tilde{W}_i^{\mathsf{T}}\phi_{i,f}$ (cf. (49) in Appendix B) and $\tilde{W}_i$ decay exponentially and $\phi_{i,f} \in \mathbb{L}_\infty$.

Next, we show that the matrix $I_{N-1}\otimes A + \alpha Q \otimes BK$ is Hurwitz given that $\alpha \geq [1/2\Re(\lambda_2(\mathcal{L}))]$. Note that any square matrix is unitarily similar to an upper triangular matrix with diagonal entries being its eigenvalues. Applying this fact to Q and noticing statement 4) of Lemma 3, it is sufficient to show that the upper triangular matrix

$$\begin{pmatrix} A+\alpha\lambda_2(\mathcal{L})BK & & \star \\ & \ddots & \\ 0 & & A+\alpha\lambda_N(\mathcal{L})BK \end{pmatrix}$$

is Hurwitz, which is then equivalent to show that the blocks on the main diagonal are Hurwitz. In fact, based on the definitions of P and K, one has

$$\begin{aligned}&(A+\alpha\lambda_i(\mathcal{L})BK)^{\mathsf{H}}P + P(A+\alpha\lambda_i(\mathcal{L})BK) \\ &\quad = A^{\mathsf{T}}P + PA^{\mathsf{T}} - \alpha\lambda_i(\mathcal{L})^{\mathsf{H}}PBB^{\mathsf{T}}P - \alpha\lambda_i(\mathcal{L})PBB^{\mathsf{T}}P \\ &\quad = AP + PA^{\mathsf{T}} - 2\alpha\Re(\lambda_i(\mathcal{L}))PBB^{\mathsf{T}}P \\ &\quad = -I_n + (1 - 2\alpha\Re(\lambda_i(\mathcal{L})))PBB^{\mathsf{T}}P\end{aligned} \tag{18}$$

for any $i \in \{2, 3, \ldots, N\}$. Then, provided $\alpha \geq [1/2\Re(\lambda_2(\mathcal{L}))]$, the blocks $A + \alpha\lambda_i(\mathcal{L})BK$ are indeed Hurwitz.

Now since $I_{N-1}\otimes A + \alpha Q\otimes BK$ is Hurwitz and e, E decay exponentially, according to statement 1 of Lemma 5, $\bar{z}$ also converges to zero exponentially (after $t \geq \max_i\{T_i\}$).

C. Exponential Consensus of x_i

Upon defining the consensus error vector $\bar{x} = (\Xi \otimes I_n)x$, one has

$$\bar{x} = \bar{z} + (\Xi\otimes I_n)e. \tag{19}$$

Then, on the basis of a)-b), it can be concluded that $\bar{x}$ converge exponentially to zero after $t \geq \max_i\{T_i\}$. Note that $\bar{x} = 0$ if and only if x_i reach consensus. This completes the proof of statement 1.

For statement 2, note from (16) that e_i and $\tilde{W}_i$ converge at the rate of $(\varrho/2)$ for all $i \in \mathcal{V}$. Then, it can be verified that e, $\tilde{W}$ and E all decay at the same rate of $(\varrho/2)$. Denote $F = I_{N-1}\otimes A + \alpha Q\otimes BK$, $F_2(t) = \alpha(\Xi\mathcal{L}\otimes BK)e + (\Xi\otimes BG)E$. Then, provided $m = n$ so that BB^{T} invertible, and the lower bound in (12), one has $\min_{k'}\{\lambda_{k'}((2\alpha\Re(\lambda_i(\mathcal{L})) - 1)PBB^{\mathsf{T}}P + I_n)\} > (\varrho/2) \quad \forall i \in \mathcal{V}$. Together with (18), the above implies that $\min_k\{\Re(\lambda_k(-F))\} > (\varrho/2)$. According to statement 2 of Lemma 5, the convergence of $\bar{z}$ in (17) is at the rate of $(\varrho/2)$. Finally, $\bar{x}$ converge at same rate of $(\varrho/2)$ due to (19). This completes the proof.

Remark 2: The closed-loop parameter identification process is integrated into the controller through modifying the inputs of the virtual reference models via E_i. It plays a key role in containing the effect aroused by the estimation error $\tilde{W}_i$, and finally leads to improved transient performance of the controller. Particularly, the filter $H(s) = (1/[ks+1])$ is introduced due to the following reasons. First, it is proper, stable and minimum-phase, preserving the PE/C-PE property of the regressors. Second, it provides an additional free parameter k for tuning: note that, as k approaches zero, the uncertainties in the closed-loop states of the agents are supposed to be completely compensated. This can be inferred by comparing the controlled state dynamics in the baseline MRACon (5)-(6) where

$$\dot{x}_i = Ax_i + BKe_i + B\gamma_i - B\tilde{W}_i^{\mathsf{T}}\phi_i \tag{20}$$

and in the improved MRACon (7)–(11) where

$$\dot{x}_i = Ax_i + BKe_i + B\gamma_i - B\tilde{W}_i^{\mathsf{T}}\big(\phi_i - \phi_{i,f}\big). \tag{21}$$

Indeed, the residual term $B\tilde{W}_i^{\mathsf{T}}(\phi_i - \phi_{i,f})$ vanishes rapidly for a small k, which implies that the modified control input can compensate the uncertainties. More quantitative discussions on the filter $H(s)$ can also be found in [28].

Remark 3: As compared to [16], a significant property of the proposed improved MRACon (7)–(11) is to guarantee exponential consensus and exponential parameter identification. Furthermore, statement 2 of Theorem 1 reveals that such exponential convergence can be attained with a determined rate provided α is sufficiently large. Intuitively, if the coupling strength among the agents is sufficiently strong so that consensus is much feasible in the network level, then the decaying rate of the closed-loop error systems will depend on the identification and tracking errors in the agent level. It is also worth remarking that the lower bound of α relies on $\Re(\lambda_2(\mathcal{L}))$, which is a global information: however, algorithms have been proposed to estimate this value in finite time and in a distributed manner [43].

IV. Adaptive Exponential Consensus With Cooperative Exponential Parameter Identification

It has been shown in [19], [20], and [21] that parameter identification in a multiagent system can be accomplished in a cooperative way, provided that the unknown parameters are homogeneous. As a byproduct, Assumption 3 can be relaxed, resulting into a weaker C-PE condition for a group of signals. In this section, we make the following assumptions.

Assumption 4: The unknown parameters in (4) are homogeneous, i.e., $W_i = W$ for all $i \in \mathcal{V}$.

Assumption 5: The digraph $\mathcal{G}$ is strongly connected and weight-balanced.

Assumption 6: The group of regressors ϕ_i is C-PE: $\{\phi_i | i \in \mathcal{I}_N\} \in$ C-PE(T, ϵ) for some $T, \epsilon \in \mathbb{R}_+$.

Remark 4: On one hand, Assumption 6 is strictly weaker than Assumption 3 [19], [20], [21]. On the other hand, although Assumption 5 is stronger than Assumption 2, it is still less restrictive than assumptions in the literature. In fact, most literature such as [19], [20], [21], [30], [31], [32], and [33] assume that $\mathcal{G}$ is undirected and connected. Assumptions 5 and 6 also appear recently in [22], however, only cooperative identification/learning is considered, without any cooperative control goal. To the best of the authors' knowledge, this is the first work on cooperative control with (cooperative) parameter identification in digraphs.

Consider the same proposed MRACon law as (7)-(10) for each agent $i \in \mathcal{V}$, with an extra consensus term involved in the adaptive law, i.e., (11) is replaced by

$$\dot{\hat{W}}_i = \Gamma\left(\phi_i e_i^{\mathsf{T}} PB + \kappa_1 S_i - \kappa_2 \sum_{j=1}^{N} a_{ij}\Big(\hat{W}_i - \hat{W}_j\Big)\right) \tag{22}$$

where $\kappa_1, \kappa_2 \in \mathbb{R}_+$. The parameter identification process is now performed in a cooperative way as compared to (11). Intuitively, each agent negotiates with his neighbors on the estimate toward the "global" homogeneous parameter, by collecting neighbors' estimates, computing consensus errors, followed by updating his own estimate accordingly.

Since the filter $H(s) = (1/[ks + 1])$ is proper, stable and minimum-phase, the following proposition can be directly stated based on Lemma 4.

Proposition 3: Under Assumption 6, the group of regressors $\phi_{i,f}$ is also C-PE, i.e., $\{\phi_{i,f} | i \in \mathcal{I}_N\} \in$ C-PE(T', ϵ') for some $T', \epsilon' \in \mathbb{R}_+$.

Besides, as an extension of statement 2) of Proposition 2, the following result can be obtained.

Proposition 4: Under Assumption 6, there exists a $\sigma' \in \mathbb{R}_+$, such that $M_s(t) \triangleq \sum_{i=1}^{N} M_i(t) \geq \sigma' I_q \quad \forall t > T'$.

Proof: See Appendix C. ■

We are now at the position to present the main result of this section.

Theorem 2: Let Assumption 1, Assumptions 4–6 hold, and choose $\alpha \geq [1/2\Re(\lambda_2(\mathcal{L}))]$ and $K = -B^{\mathsf{T}}P$ as in Proposition 1. Then, with the proposed improved MRACon (7)–(10) and (22), it holds that

1) the agents (3) achieve exponential consensus with parameter identification, where T^* in Definition 4 can be specified as $T^* = T'$ (defined in Proposition 3);
2) if $m = n$, let α further satisfy
$$\alpha > \frac{1}{2\Re(\lambda_2(\mathcal{L}))} + \frac{\varrho' - 2}{4\Re(\lambda_2(\mathcal{L}))\lambda_{\min}\big(PBB^{\mathsf{T}}P\big)} \tag{23}$$
where $\varrho' \triangleq \min\{([1 + \lambda_{\min}(PBB^{\mathsf{T}}P)]/\lambda_{\max}(P)), 2\kappa_3 \eta\lambda_{\min}(\Gamma)\} > 0$ with $\kappa_3 \triangleq \min\{\kappa_1, \kappa_2\}$ and η as defined in (34), then, the exponential convergence in both consensus and parameter identification is at the rate of $(\varrho'/2)$.

Proof: Note that statement 1) of Proposition 2 still holds, i.e., $S = -M_{\mathrm{d}}\tilde{W}$ where $S = \mathrm{col}(S_1, \ldots, S_N)$, $M_{\mathrm{d}} = \mathrm{diag}(M_1, \ldots, M_N)$, and $\tilde{W} = \mathrm{col}(\tilde{W}_1, \ldots, \tilde{W}_N)$. Furthermore, define $\tilde{W}_{\mathrm{d}} = \mathrm{diag}(\tilde{W}_1, \ldots, \tilde{W}_N)$, $\phi = \mathrm{col}(\phi_1, \ldots, \phi_N)$. Then, based on (45) and (22), the closed-loop error system consisting of virtual tracking errors and identification errors can be written in a compact form as

$$\begin{aligned}\dot{e} &= \big(I_N \otimes (A + BK)\big)e - (I_N \otimes B)\tilde{W}_{\mathrm{d}}^{\mathsf{T}}\phi \\ \dot{\tilde{W}} &= (I_N \otimes \Gamma)\Big(\mathrm{diag}\Big(\phi_1 e_1^{\mathsf{T}}, \ldots, \phi_N e_N^{\mathsf{T}}\Big)(\mathbf{1}_N \otimes PB) \\ &\quad - \big(\kappa_1 M_{\mathrm{d}} + \kappa_2\big(\mathcal{L} \otimes I_q\big)\big)\tilde{W}\Big).\end{aligned} \tag{24}$$

Similar to the proof of Theorem 1, we first aim to prove the exponentially convergence of the error system (24) under Assumption 6. Let us consider the candidate Lyapunov function for the error system (24) as

$$V\big(e, \tilde{W}\big) = e^{\mathsf{T}}(I_N \otimes P)e + \mathrm{tr}\Big(\tilde{W}^{\mathsf{T}}\Big(I_N \otimes \Gamma^{-1}\Big)\tilde{W}\Big). \tag{25}$$

Analyzing the time derivative of V along the trajectories of (24), followed by some manipulations similar to (15), yields

$$\begin{aligned}\dot{V} = &-e^{\mathsf{T}}\Big(I_N \otimes \Big(I_n + PBB^{\mathsf{T}}P\Big)\Big)e \\ &-2\mathrm{tr}\Big(\tilde{W}^{\mathsf{T}}\big(\kappa_1 M_{\mathrm{d}} + \kappa_2\big(\mathcal{L} \otimes I_q\big)\big)\tilde{W}\Big).\end{aligned} \tag{26}$$

Under Assumption 5, $\tilde{W}^\mathsf{T}(\mathcal{L}\otimes I_q)\tilde{W} = \tilde{W}^\mathsf{T}(\hat{\mathcal{L}}\otimes I_q)\tilde{W}$ where $\hat{\mathcal{L}}$ is defined as in Lemma 2 with $r_0 = (1/2)$. It is clear that M_d and $\hat{\mathcal{L}}$ are both positive semidefinite. Then

$$\begin{aligned}\dot{V} \leq & -e^\mathsf{T}\Big(I_N \otimes \Big(I_n + PBB^\mathsf{T}P\Big)\Big)e \\ & -2\kappa_3 \mathrm{tr}\Big(\tilde{W}^\mathsf{T}\Big(M_\mathrm{d} + \Big(\hat{\mathcal{L}}\otimes I_q\Big)\Big)\tilde{W}\Big)\end{aligned} \tag{27}$$

where $\kappa_3 = \min\{\kappa_1, \kappa_2\}$. Next, we show that the matrix $M_\mathrm{d}(t) + (\hat{\mathcal{L}}\otimes I_q)$ is uniformly positive definite $\forall t \geq T'$ with T' defined in Proposition 4.

According to Lemma 2, the matrix $\hat{\mathcal{L}}$ has only one zero eigenvalue whose unit eigenvectors is $(1/\sqrt{N})\mathbf{1}_N$. Then, the matrix $\hat{\mathcal{L}} \otimes I_q$ has q eigenvalues and the corresponding orthogonal unit eigenvectors are

$$v_1 = \frac{1}{\sqrt{N}}\mathbf{1}_N \otimes \pi_1,\ \ldots,\ v_q = \frac{1}{\sqrt{N}}\mathbf{1}_N \otimes \pi_q \tag{28}$$

where $\pi_i \triangleq (0,\ldots,1,\ldots,0)^\mathsf{T} \in \mathbb{R}^q$ (the ith element being 1), $i \in \mathcal{I}_q$, form a basis of $\mathbb{R}^q$. Denote the other eigenvalues of $\hat{\mathcal{L}}\otimes I_q$ as $0 < \lambda_{q+1} \leq \lambda_{q+2} \leq \cdots \leq \lambda_{Nq}$, and denote the corresponding orthogonal unit eigenvectors as $v_{q+1},\ldots,v_{Nq}$. For any nonzero vector $\xi \in \mathbb{R}^{Nq}$, it can be represented by

$$\xi = \sum_{i=1}^{q} c_i v_i + \sum_{i=q+1}^{Nq} c_i v_i \tag{29}$$

with a unique group of c_i, $i = 1,\ldots,Nq$, which cannot be zero simultaneously. On one hand, when $\sum_{i=q+1}^{Nq} c_i^2 > 0$, one has

$$\begin{aligned}\xi^\mathsf{T}\Big(M_\mathrm{d} + \Big(\hat{\mathcal{L}}\otimes I_q\Big)\Big)\xi &= \xi^\mathsf{T} M_\mathrm{d}\xi + \sum_{i=q+1}^{Nq} c_i^2\lambda_i \\ &\geq \sum_{i=q+1}^{Nq} c_i^2 \lambda_{q+1} > 0.\end{aligned} \tag{30}$$

On the other hand, when $\sum_{i=q+1}^{Nq} c_i^2 = 0$, one has $\sum_{i=1}^{q} c_i^2 > 0$. Then

$$\begin{aligned}\xi^\mathsf{T}\Big(M_\mathrm{d} + \Big(\hat{\mathcal{L}}\otimes I_q\Big)\Big)\xi &= \left(\sum_{i=1}^{q} c_i v_i\right)^\mathsf{T} M_\mathrm{d}\left(\sum_{i=1}^{q} c_i v_i\right) \\ &= C^\mathsf{T} V^\mathsf{T} M_\mathrm{d} V C\end{aligned} \tag{31}$$

where $C = (c_1,\ldots,c_q)^\mathsf{T}$ and $V = (v_1,\ldots,v_q)$. It can be verified that

$$V^\mathsf{T} M_\mathrm{d} V = \frac{1}{N}\sum_{i=1}^{N} M_i = \frac{1}{N} M_\mathrm{s}. \tag{32}$$

According to Proposition 4, one has $\forall t > T'$

$$\begin{aligned}\xi^\mathsf{T}\Big(M_\mathrm{d} + \Big(\hat{\mathcal{L}}\otimes I_q\Big)\Big)\xi &= \frac{1}{N} C^\mathsf{T} M_\mathrm{s} C \\ &\geq \frac{\sigma'}{N}\sum_{i=1}^{q} c_i^2 > 0.\end{aligned} \tag{33}$$

Summarizing (28)-(33), we have shown that the matrix $M_\mathrm{d} + (\hat{\mathcal{L}}\otimes I_q)$ $\forall t \geq T'$, is positive definite. Since M_i is smooth and $\hat{\mathcal{L}}$ is fixed, one can use similar contradiction arguments as in [19] and [32] to show that the matrix $M_\mathrm{d} + (\hat{\mathcal{L}}\otimes I_q)$ is uniformly positive definite, i.e.,

$$M_\mathrm{d}(t) + \Big(\hat{\mathcal{L}}\otimes I_q\Big) \geq \eta I_q \qquad \forall t \geq T' \tag{34}$$

for some $\eta \in \mathbb{R}_+$. Then, it follows from (27) and (34) that:

$$\dot{V}(t) \leq -\varrho' V(t) \qquad \forall t \geq T'. \tag{35}$$

As such, e and $\tilde{W}$ converge exponentially to zero after $t \geq T'$.

Consequently, following similar steps b)-c) in the proof of Theorem 1, one can show that x_i also reach consensus exponentially after $t \geq T'$, thus statement 1) holds. Finally, statement 2) can be proved in a similar way as in the proof of Theorem 1. The proof is completed. ∎

Remark 5: Similar to [44], the final consensus manifold is allowed to be manipulated by including some global external reference inputs in $\tilde{\gamma}_i$ in (7). Besides, one can also include the same state feedback loop to the agents and the reference models simultaneously to specify some desired consensus manifolds, e.g., to track a global reference model (cf. Example 2 in Section V). The proofs of Theorem 1 and 2 still applies to these extensions without much modifications, and thus, the details are omitted.

Remark 6: Recent advances on parameter identification have shown that the PE condition is not necessary for identification and can be relaxed, see, e.g., IE in single-agent systems [23], [24], [25], [26] and, likewise, cooperative IE (C-IE) in multiagent systems [31], [32], [33]. In principle, the results proposed in this article could be extended under IE/C-IE conditions and two possible ways are envisaged to do this. The first way is to use integrator filters, i.e., to set the forgetting factor $l = 0$ in the filters (10), which resembles the integrator filters in [23] and [32]: here, robustness issues may arise due to the monotonically increasing M_i. The second way is to incorporate appropriate switching mechanisms in the parameter adaptation law as in [31] and [33], which may nevertheless induce nonsmoothness into the controller. For these reasons, the results of this work have been presented under PE/C-PE conditions, leaving the IE/C-IE cases for future work. It is also worth remarking that when the forgetting factor $l \to 0$, the required excitation conditions approach IE/C-IE conditions, respectively [28].

V. Simulations

Example 1: Consider a network of $N = 6$ uncertain second-order agents over a directed ring graph with $\mathcal{E} = \{\mathcal{E}_{12}, \mathcal{E}_{23}, \mathcal{E}_{34}, \mathcal{E}_{45}, \mathcal{E}_{56}, \mathcal{E}_{61}\}$. The dynamics of the agents follows (3) with:

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},\ B = I_2,\ f_i = W^\mathsf{T}\phi_i \tag{36}$$

where $W^\mathsf{T} = \begin{pmatrix} 2 & -1 & 0 \\ -5 & 0 & 10 \end{pmatrix}$ is unknown to the agents, and different choices for ϕ_i will be specified later on, so as to satisfy different PE conditions. In line with Assumption 4, W is homogeneous for all agents. In the simulations, the initial x_i, z_i and initial estimation $\hat{W}_i$ for the agents are randomly selected according to the Gaussian distribution. The coupling gain is

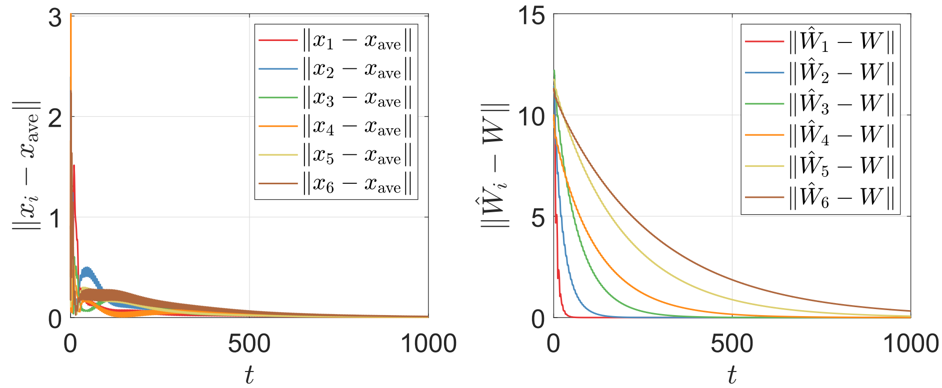

Fig. 3. Case 1 (PE): The consensus errors and parameter identification errors of the agents with the baseline MRACon (5)-(6) proposed in [16].

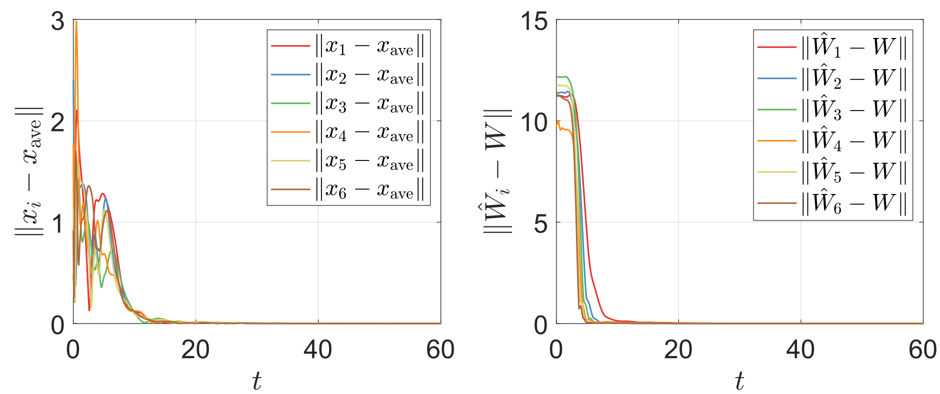

Fig. 4. Case 1 (PE): The consensus errors and parameter identification errors of the agents with the proposed improved MRACon (7)–(11).

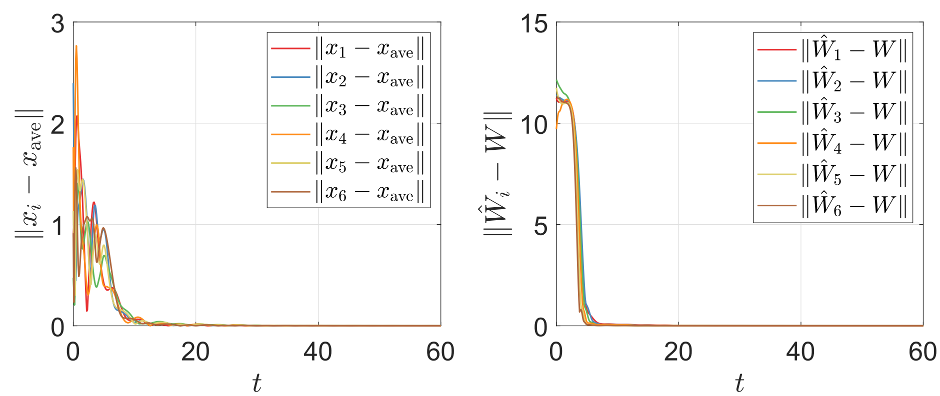

Fig. 5. Case 1 (PE): The consensus errors and parameter identification errors of the agents with the proposed improved MRACon (7)–(10) and (22).

selected as the lower bound $\alpha = 1$ since $\Re(\lambda_2(\mathcal{L})) = 0.5$ for the ring topology. Let $\Gamma = I_3$ for the adaptation laws (6), (11) and (22). We define $x_{\text{ave}} = (1/N)\sum_{i=1}^{N} x_i$.

Case 1: The regressor ϕ_i is PE $\forall i \in \mathcal{V}$;

Suppose that $\phi_i = (i\sin(t), i\cos(t), [it/(t+1)])^{\mathsf{T}}$, $i \in \mathcal{V}$, then it is clear that each ϕ_i satisfies Definition 1. The following Figs. 3–5 shows the simulation results with the baseline MRACon [16] and two proposed improved MRACon laws in Sections III and IV. The parameters used in (7)-(11) are $k = 0.5$, $l = 0.1$, $\delta = 0.5$, $\kappa_1 = 1$. For the cooperative identification case (7)-(10) with (22), we further use $\kappa_2 = 1$.

Case 2: The group of regressors ϕ_i, $i \in \mathcal{V}$, is C-PE;

Let $\phi_i = \exp^{-t}(i\sin(t), i\cos(t), [it/(t+1)])^{\mathsf{T}}$, $i \in \mathcal{V} - \{6\}$, and $\phi_6 = (6\sin(t), 6\cos(t), [6t/(t+1)])^{\mathsf{T}}$. Then, the fact that only ϕ_6 is PE is still sufficient to make the group of ϕ_i, $i \in \mathcal{V}$ satisfy Definition 2. The following Figs. 6–8 shows the simulation results with the three methods using the same set of parameters as in Case 1.

An immediate observation is that both proposed improved MRACon laws accelerate the leaderless consensus of the agents drastically in both cases (notice the difference in time scales). The credit should be given to the idea of feedback, i.e., to close the loop of parameter estimates, which allows to conclude exponentially convergence of the consensus errors. It is worth remarking that the performance improvements

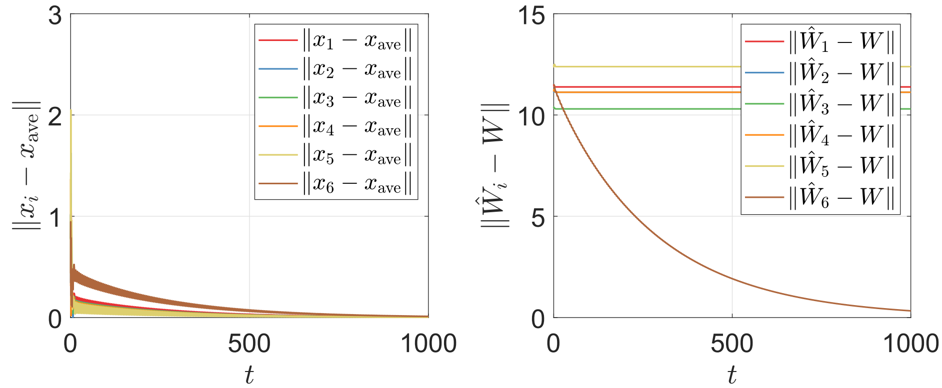

Fig. 6. Case 2 (C-PE): The consensus errors and parameter identification errors of the agents with the baseline MRACon (5)-(6) proposed in [16].

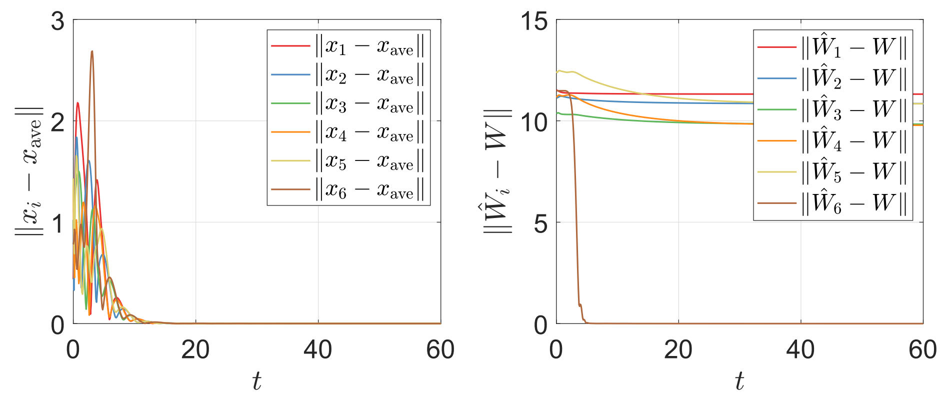

Fig. 7. Case 2 (C-PE): The consensus errors and parameter identification errors of the agents with the proposed improved MRACon (7)–(11).

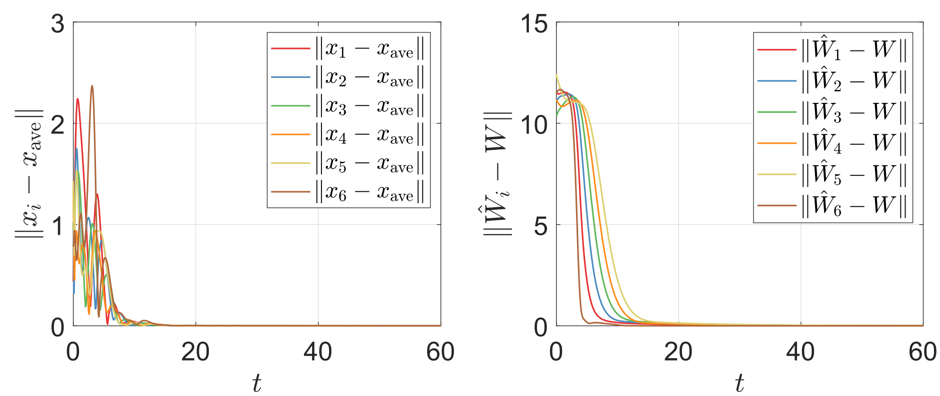

Fig. 8. Case 2 (C-PE): The consensus errors and parameter identification errors of the agents with the proposed improved MRACon (7)-(10) and (22).

are barely at the cost of more control efforts. For instance, computing the integral $\int_0^{60} \|u_6(\tau)\| \mathrm{d}\tau$ in case 2 across all three methods returns 3354.5, 3356.7 and 3359.2, respectively, for the baseline and the two improved MRACon laws. This highlights the efficiency of the proposed framework.

The second observation is about the parameter identification errors. It can be seen by comparing Figs. 4 and 3 that the improved MRACon law (7)–(11) leads to much faster parameter identification when each regressor ϕ_i is PE. Also, one can see from Fig. 7 that, when the group of regressors is C-PE, the improved MRACon law (7)–(11) fails the task of parameter identification for all agents, and only agent 6 with the PE regressor identifies the real parameter as expected. In this case, as shown in Fig. 8, the improved MRACon (7)–(10) with the cooperative identification law (22) allows exact parameter identification for all agents in the network system. The simulation results agree with theoretical results.

Example 2: In this example, we show how the proposed methods can be applied to wing rock aircraft group control. Consider a network of $N = 6$ aircrafts over the same directed ring graph as in Example 1. The model of the aircrafts follows from [28] and [45] as:

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \; B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \; f_i = W^{\mathsf{T}} \phi_i \qquad (37)$$

where f_i represents the parameterized aerodynamics with regressor $\phi_i = (x_{i1}, x_{i2}, |x_{i1}|x_{i2}, |x_{i2}|x_{i2}, x_{i1}^3)^{\mathsf{T}}$, and coefficient $W^{\mathsf{T}} = (0.2314, 0.6918, -0.6254, 0.0095, 0.0214)$: this true

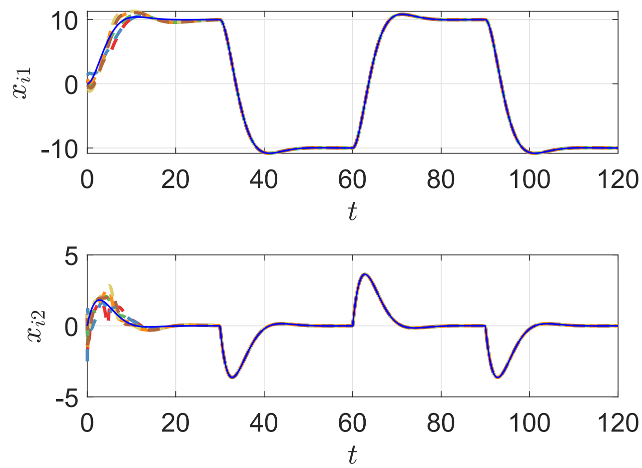

Fig. 9. States of the aircrafts with the proposed improved MRACon.

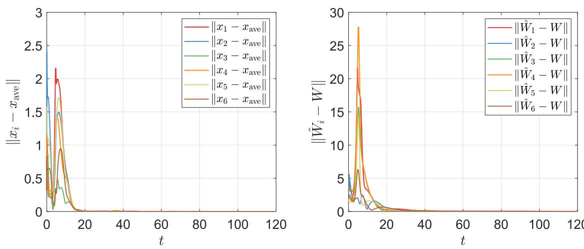

Fig. 10. Consensus errors and parameter identification errors of the aircrafts with the proposed improved MRACon.

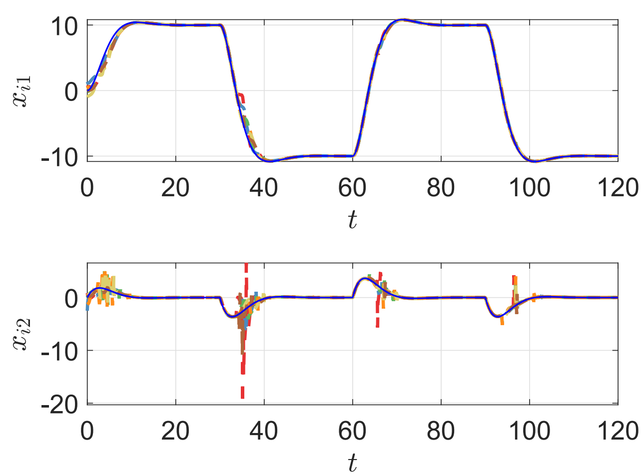

Fig. 11. States of the aircrafts with the baseline MRACon.

value of W is unknown for control design. The state x_{i1} and x_{i2} represent the roll angle and roll rate of aircraft i, respectively, and a single input controls the aileron.

The control goal is such that the roll dynamics of (37) for all aircrafts can reach consensus, meanwhile, can track the trajectory of an ideal global reference model with a natural circular frequency $\omega = 0.4$ rad/sec, a damping ratio $\zeta = 0.707$, with a global reference input r being a square wave of period 60 s and amplitude 20 rad. This leads to a nominal state feedback action with state gain $K_{\text{nom}} = (-0.16, -0.57)$, and reference input gain $K_{\text{r}} = 0.16$, applied to both x and z loops. The proposed improved MRACon (8)–(10) and (22), along with the nominal state feedbacks, is applied to each aircraft. The parameters used are $k = 0.01$, $l = 0.1$, $\delta = 0.3$, $\kappa_1 = 0.5$, $\kappa_2 = 0.1$, $\Gamma = \text{diag}(1, 3, 1, 1, 0.001)$. The following Figs. 9 and 10 show the simulation results where the consensus tracking goal is achieved, and the real parameter is successfully identified by all each aircraft. We also simulate the baseline MRACon under the same initial conditions, and the results are shown in Figs. 11 and 12. It can be seen that the baseline MRACon lacks robustness for consensus reference tracking, and more importantly, fails to identify the unknown parameters.

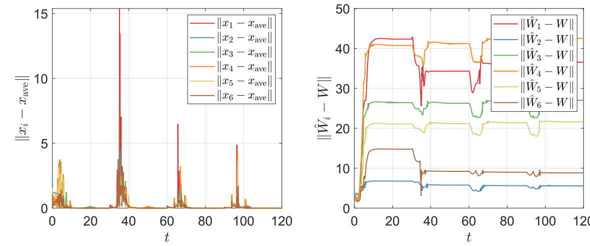

Fig. 12. Consensus errors and parameter identification errors of the aircrafts with the baseline MRACon.

Algorithm 1: Improved MRACon With Cooperative Parameter Identification for Aircraft Control (Ex. 2)

Data: (1) Initialize x_i, z_i, $\hat{W}_i$ randomly and $e_{i,f}$, $\varepsilon_{i,f}$, $\phi_{i,f}$, M_i, S_i as zero
(2) Preset the reference input r, discrete-time step h, terminal time T_{tml}
(3) Set parameters k, l, δ, κ_1, κ_2, Γ, K_{nom}, K_{r}
Result: Track to the preset reference model with parameter identification, for each aircraft

Solve α and K;
$\tau \leftarrow 1$;
while $\tau \cdot h \leq T_{tml}$ **do**
  for $i \leftarrow 1$ **to** N **do**
    Solve $u_i = Ke_i - \hat{W}_i^{\top}\phi_i + \tilde{\gamma}_i + K_{\text{nom}}x_i + K_{\text{r}}r$;
    Update $e_{i,f}, \varepsilon_{i,f}, \phi_{i,f}, M_i, S_i$ via (9)-(10);
    Update z_i via $\dot{z}_i = Az_i + B(\tilde{\gamma}_i + K_{\text{nom}}z_i + K_{\text{r}}r)$;
    Update $\hat{W}_i$ via (22);
    Update x_i via (3);
  end
  $\tau \leftarrow \tau + 1$;
end

To close the section, the pseudo code for the proposed algorithm (7)–(10) and (22), applied to this aircraft control problem, is provided in Algorithm 1 for better readability.

VI. Conclusion

The problem of leaderless consensus with parameter identification has been addressed for multiagent systems with possibly heterogeneous parametric uncertainties over directed graphs. Two novel improvements on the existing MRACon solution has been proposed. The first improvement is inspired by the idea of closed-loop parameter identification, and the second improvement further expands the proposed framework in the sense of cooperative parameter identification. Both proposed improvements guarantee exponential convergence of the consensus errors and parameter identification errors. Some interesting future topics are extensions in the directions of finite/fixed-time stability, event-triggered communication, and real-world robot applications such as quadcopters, manipulators, etc.

APPENDIX A
PROOF OF LEMMA 4

Denote the filtered signals as $\tilde{\phi}_i = G(s)\phi_i$. The boundedness of each $\tilde{\phi}_i$ is evident since $G(s)$ is proper stable and ϕ_i is bounded (see [18, Lemma 4.8.2]). In the following, we prove that the group of $\tilde{\phi}_i$ is C-PE, i.e., to prove that:

$$\int_t^{t+T} \sum_{i=1}^{N} \tilde{\phi}_i(\tau)\tilde{\phi}_i(\tau)^{\mathsf{T}} \mathrm{d}\tau \geq \epsilon I \qquad \forall t \geq 0 \tag{38}$$

and for some $T, \epsilon \in \mathbb{R}_+$. In the scalar form, (38) is equivalent to

$$\int_t^{t+T} \sum_{i=1}^{N} \left(\xi^{\mathsf{T}}\tilde{\phi}_i(\tau)\right)^2 \mathrm{d}\tau \geq \epsilon \qquad \forall t \geq 0 \tag{39}$$

where $\xi \in \mathbb{R}^n$ is any unit vector with $\|\xi\| = 1$.

Denote $\eta_i = \xi^{\mathsf{T}}\phi_i$ and $\tilde{\eta}_i = G(s)\eta_i$. Then, $\tilde{\eta}_i = \xi^{\mathsf{T}}\tilde{\phi}_i$ since ξ does not depend on t. This translates (39) into

$$\int_t^{t+T} \sum_{i=1}^{N} \tilde{\eta}_i^2(\tau)\mathrm{d}\tau \geq \epsilon \qquad \forall t \geq 0 \tag{40}$$

for some $T, \epsilon \in \mathbb{R}_+$. On the other side, the condition that ϕ_i is C-PE is equivalent to

$$\int_t^{t+T_0} \sum_{i=1}^{N} \eta_i^2(\tau)\mathrm{d}\tau \geq \epsilon_0 \qquad \forall t \geq 0 \tag{41}$$

for some $T_0, \epsilon_0 \in \mathbb{R}_+$. The problem appears on the relationship between accumulated truncated $\mathbb{L}_2$ norms of multiple inputs and respect outputs of a stable, minimum-phase filter.

Define $z_i = [a^r/(s+a)^r]\eta_i$ where $a \in \mathbb{R}_+$ will be specified later and r is the relative degree of $G(s)$. Then

$$z_i = \frac{a^r}{(s+a)^r}G^{-1}(s)\tilde{\eta}_i.$$

It is clear that the transfer function from $\tilde{\eta}_i$ to z_i is proper and stable. Then, based on [18, Lemma 4.8.2], there exist some $k_{i1}, k_{i2} \in \mathbb{R}_+$ which may depend on a, such that

$$\int_t^{t+T} z_i^2(\tau)\mathrm{d}\tau \leq k_{i1}\int_t^{t+T} \tilde{\eta}_i^2(\tau)\mathrm{d}\tau + k_{i2}$$

hold for any $t, T \in \mathbb{R}_+$. On the other hand, according to the proof of [18, Lemma 4.8.3], there exist some $k_{i3}, k_{i4} \in \mathbb{R}_+$ such that

$$\int_t^{t+T} \eta_i^2(\tau)\mathrm{d}\tau \leq 2\int_t^{t+T} z_i^2(\tau)\mathrm{d}\tau + \frac{k_{i3}}{a^2}T + k_{i4}$$

where k_{i3} depends on the upper bound of $\dot{\phi}_i$ and k_{i3}, k_{i4} are independent of a.

Let $k_j = \max_{i\in\mathcal{V}}\{k_{ij}\}$, $j = 1, 2, 3, 4$. Then, for all $i \in \mathcal{V}$ and any $t, T \in \mathbb{R}_+$

$$\int_t^{t+T} z_i^2(\tau)\mathrm{d}\tau \leq k_1\int_t^{t+T} \tilde{\eta}_i^2(\tau)\mathrm{d}\tau + k_2, \tag{42}$$

$$\int_t^{t+T} \eta_i^2(\tau)\mathrm{d}\tau \leq 2\int_t^{t+T} z_i^2(\tau)\mathrm{d}\tau + \frac{k_3}{a^2}T + k_4. \tag{43}$$

Summarizing over $i \in \mathcal{V}$ in (42) and (43) yields

$$\begin{aligned}\int_t^{t+T} \sum_{i=1}^{N} \tilde{\eta}_i^2(\tau)\mathrm{d}\tau &\geq \frac{1}{k_1}\left(\int_t^{t+T} \sum_{i=1}^{N} z_i^2(\tau)\mathrm{d}\tau - Nk_2\right)\\ &\geq \frac{1}{k_1}\Big(\frac{1}{2}\sum_{i=1}^{N}\int_t^{t+T} \eta_i^2(\tau)\mathrm{d}\tau\\ &\quad - \frac{Nk_3}{a^2}T - Nk_2 - Nk_4\Big).\end{aligned} \tag{44}$$

Let $T = mT_0$ where T_0 is defined in (41) and m is a positive integer to be specified later. Then, it follows from (41) and (45) that $\forall t \geq 0$:

$$\begin{aligned}\int_t^{t+T} \sum_{i=1}^{N} \tilde{\eta}_i^2(\tau)\mathrm{d}\tau &\geq \frac{1}{k_1}\left(\frac{m\epsilon_0}{2} - \frac{Nk_3}{a^2}T - Nk_2 - Nk_4\right)\\ &= \frac{1}{k_1}\left(m\left(\frac{\epsilon_0}{2} - \frac{Nk_3T_0}{a^2}\right) - Nk_2 - Nk_4\right).\end{aligned}$$

Since k_3 is independent of a, one can always select sufficiently large a such that $(\epsilon_0/2) - [Nk_3T_0/a^2] \geq (\epsilon_0/4)$. Meanwhile, let m be sufficiently large such that $m(\epsilon_0/4) - Nk_2 - Nk_4 \geq \epsilon_0$. Then, the lower bound (40) is satisfied with $\epsilon = (\epsilon_0/k_1)$. In other words, $\{\tilde{\phi}_i | i \in \mathcal{I}_N\} \in$ C-PE$(mT_0, [\epsilon_0/k_1])$. The proof is completed.

APPENDIX B
PROOF OF PROPOSITION 2

Substituting (4) and (7) into (3) yields

$$\dot{e}_i = (A + BK)e_i - B\tilde{W}_i^{\mathsf{T}}\phi_i. \tag{45}$$

Applying the filter $H(s)$ to both sides of (45) yields

$$\dot{e}_{i,f} = (A + BK)e_{i,f} - BH(s)\left(\tilde{W}_i^{\mathsf{T}}\phi_i\right). \tag{46}$$

By swapping lemma [18, Lemma A.1]

$$\begin{aligned}H(s)\left(\tilde{W}_i^{\mathsf{T}}\phi_i\right) &= \tilde{W}_i^{\mathsf{T}}H(s)\phi_i - \frac{k}{ks+1}\left(\dot{\tilde{W}}_i^{\mathsf{T}}H(s)\phi_i\right)\\ &= \tilde{W}_i^{\mathsf{T}}\phi_{i,f} - H(s)\left(k\dot{\hat{W}}_i^{\mathsf{T}}\phi_{i,f}\right)\\ &= \tilde{W}_i^{\mathsf{T}}\phi_{i,f} - \varepsilon_{i,f}.\end{aligned} \tag{47}$$

Then

$$\dot{e}_{i,f} = (A + BK)e_{i,f} - B\tilde{W}_i^{\mathsf{T}}\phi_{i,f} + B\varepsilon_{i,f}. \tag{48}$$

Note that $\dot{e}_{i,f} = ([e_i - e_{i,f}]/k)$ by design. Then, it follows from (48) and the definition of E_i that:

$$E_i = B\tilde{W}_i^{\mathsf{T}}\phi_{i,f}. \tag{49}$$

Solving M_i in (8) gives

$$M_i = \delta\int_0^t e^{-l(t-\tau)}\phi_{i,f}\phi_{i,f}^{\mathsf{T}}\mathrm{d}\tau. \tag{50}$$

Then, substituting (49)-(50) into the dynamics of S_i yields

$$\dot{S}_i = -lS_i - \delta\phi_{i,f}\phi_{i,f}^{\mathsf{T}}\tilde{W}_i - \delta\int_0^t e^{-l(t-\tau)}\phi_{i,f}\phi_{i,f}^{\mathsf{T}}\mathrm{d}\tau\,\dot{\hat{W}}_i. \quad (51)$$

Left-multiplying e^{lt} to both sides of (51), followed by some manipulations, gives the complete differential form:

$$\frac{\mathrm{d}\left(e^{lt}S_i\right)}{\mathrm{d}t} = \frac{-\delta\mathrm{d}\left(\tilde{W}_i\int_0^t e^{l\tau}\phi_{i,f}\phi_{i,f}^{\mathsf{T}}\mathrm{d}\tau\right)}{\mathrm{d}t}. \quad (52)$$

After taking the integral of (52), one has

$$e^{lt}S_i = -\delta\tilde{W}_i\int_0^t e^{l\tau}\phi_{i,f}\phi_{i,f}^{\mathsf{T}}\mathrm{d}\tau + C \quad (53)$$

for a constant C. Note that $C = 0$ since $S_i(0) = 0$. Then, $S_i = -M_i\tilde{W}_i$ holds. The proof of statement 2) can be found in [28], thus it can be omitted.

Appendix C
Proof of Proposition 4

From (50), one has

$$M_{\mathrm{s}} = \delta\sum_{i=1}^{N}\int_0^t e^{-l(t-\tau)}\phi_{i,f}\phi_{i,f}^{\mathsf{T}}\mathrm{d}\tau. \quad (54)$$

Then $\forall t \geq T'$

$$\begin{aligned} M_{\mathrm{s}} &\geq \delta\sum_{i=1}^{N}\int_{t-T'}^{t} e^{-l(t-\tau)}\phi_{i,f}\phi_{i,f}^{\mathsf{T}}\mathrm{d}\tau \\ &\geq \delta e^{-lT'}\int_{t-T'}^{t}\sum_{i=1}^{N}\phi_{i,f}\phi_{i,f}^{\mathsf{T}}\mathrm{d}\tau \geq \delta\epsilon' e^{-lT'}I_q. \end{aligned} \quad (55)$$

where the fact that $e^{-l(t-\tau)} \geq e^{-lT'}$ for any $\tau \in [t-T', t]$ and Proposition 3 have been used to obtain the second and the last inequality, respectively. Then, Proposition 4 holds with $\sigma' = \delta\epsilon' e^{-lT'}$.

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