Distributed Neuro-Adaptive Formation Control for Uncertain Multi-Agent Systems: Node- and Edge-Based Designs

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***Abstract*—Distributed neuro-adaptive Time-Varying Formation (TVF) control for multi-agent systems with matching unknown nonlinearities is considered. According to different perspectives of the dynamical coupling strengths between the agents, two control strategies, named node- and edge-based, are designed and analyzed in the framework of Lyapunov theory, respectively. With the help of neural networks and nonsmooth analysis, both controllers guarantee the robust asymptotical convergence of the TVF errors and can also resist unknown matching disturbances. Node-based design is found to be fully-distributed, which does not depend on any global information, meanwhile the edge-based design is applicable for TVF on switching graphs. Some numerical simulations are provided to support the theoretical results.**

***Index Terms*—adaptive control, formation control, multi-agent systems, neural network.**

I. Introduction

OVER the past two decades, there have been significant research efforts focusing on Multi-Agent Systems (MASs) due to their practical as well as theoretical values. Practically, MASs can be used to model various network systems in industry, engineering, sports, such as multiple robots with sensing and acting capabilities, vehicles in traffic, players in synchronized swimming, etc. Theoretically, to discover inner mechanisms between the individuals and the community is challenging as well as interesting itself. For instance, fully-distributed control [1]–[3], i.e., controlling the individuals without a need of any global information or central commander, and control of switching network system [4] are two of the research hot spots recently.

Formation control, as one of the most active fields in MAS community, generally aims to drive multiple agents to span some geometrical shapes. Depending on the inherent dynamics and the constrains on the network, various of formation control problems have been studied in the literature, see [5]–[8] and the references therein. Based on different formation tasks, the studies on formation can be roughly divided into the time-invariant formation, Time-Varying Formation (TVF) and Time-Varying Formation Tracking (TVFT) [9]. Consensus-based formation control [10] is an established methodology to accomplish these formation tasks in a distributed way.

On the one hand, the consensus-based formation control has been well understood for Linear Time-Invariant MASs (LTI-MASs). For instance, formation control problem has been addressed for first-order MASs in [9] and general LTI-MASs in [11], both with finite-time convergence properties, and second-order MASs in [12] with communication delays. TVF protocols have been considered for high-order LTI-MASs in [13] in the presence of time delays, and in [14] with switching digraphs. Thereafter, TVFT with multiple leader agents has been analyzed in [15]. Robust formation problems for discrete time LTI-MASs subject to stochastic disturbances and model uncertainties have been addressed in [16] through an iterative learning approach. Cluster formation for LTI-MASs under aperiodic sampling and communication delays has been developed in [17]. Fully-distributed adaptive TVF can be found recently in [18], [19].

On the other hand, the studies on formation control of nonlinear MASs are far from being rich and some relevant works are simply surveyed in the following. For first-order nonlinear MASs, [20] has studied the formation control problem via the iterative learning control. [21] has studied the optimized formation control by using fuzzy logic systems and reinforcement learning. For second-order nonlinear MASs, Neural Network (NN) based adaptive output feedback formation control with unknown nonlinear functions and bounded disturbances has been considered in [22]. NN-based TVFT with multiple leaders has been studied in [23]. When the nonlinearities are locally Lipschitz, a class of nonsmooth leader-following asymptotic formation controller has been designed in [24]. For higher-order nonlinear systems, [25] has considered the TVFT problem with

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Lipschitz nonlinearity and unknown disturbances by designing disturbance observers. Recently, [26] has addressed the robust TVFT problem for a general class of high-order nonlinear MASs with matching bounded model uncertainties and disturbances, whereas the design process is dependent on the communication topologies, thus it is not fully-distributed.

It is well known that one advantage of adaptive control in comparison with robust control is that the no boundness information is required on the model uncertainties. By leveraging the great nonlinear function approaching capability of the NNs, [27]–[30] have recently investigated the leader-follower consensus control problems for unknown first-order, second-order, higher-order and matching nonlinear MASs, respectively. More recently in [31], the asymptotic neuro-adaptive containment control of MASs with unmodelled dynamics has been realized by defining a novel pseudo ideal weight matrix for each agent. Even though, the results in these aforementioned contributions are not fully-distributed either, as the underlying Laplacian spectrums are still required when designing the protocols. The extension of the NN-based method to adaptive TVF problem for nonlinear MASs is open, partly motivating the study of this work.

The state-of-the-art adaptive TVF problems in the literature can be roughly divided into two classes. For the first class, the purpose of adaptive control is to approximate the uncertainties in the dynamics of the agents, see [22], [23] for example. These works often rely on NNs, fuzzy logic or other intelligent control tools, yet one drawback is that the design process often requires the knowledge of the Laplacian eigenvalues. For the second class, the purpose of adaptive control is to decouple the controller design with the network Laplacian, see [18], [19] for example. These works focus on designing suitable time-varying coupling weights, leading to fully-distributed schemes, while the dynamics of the agents are often well understood with less uncertainties. An interesting problem is whether the advantages of the above two strategies can be merged together, and the study of this work give a positive answer. This paper studies the NN-based distributed adaptive TVF control for a class of MASs with matching unknown nonlinearities. Here the nonlinearities are either suppressed by known functions, nor assumed to be Lipschitz continuous thanks to the capability of the NNs to approximate nonlinear functions. The main contributions can be summarized into the following four aspects:

- In comparison with adaptive TVF problems in [18], [19], the considered agent dynamics are not LTI, but more general nonlinear, which is more challenging.
- The results extend those of [30], [32], [33] from consensus to formation control.
- In comparison with [22], [23], [30], [31], node-based fully-distributed controller for uncertain MASs is developed by incorporating adaptive strategies in the coupling weights, which enhances the flexibility for control and convenience for maintenance.
- By introducing the concept of pseudo ideal weight matrix [31], Semi-Global Asymptotic Time-Varying Formation (SGA-TVF) is achieved, while traditional NN-based analysis in [27]–[30] leads to Semi-Global Ultimately Uniformly Bounded (SGUUB) residual errors.

The remaining of this paper is organized as follows. In Section II, some preliminaries and problem statement are formulated. In Section III, main results are presented through two subsections focusing on the node- and edge—based designs respectively. In Section IV, two different TVF instances are simulated and analysed with the proposed controllers. Section V finally summarizes the whole paper and discusses potential topics in the future.

II. Preliminaries and Problem Statement

A. Notations, Graph Theory, and Useful Lemmas

In this paper, $\mathbb{R}$ (resp. $\mathbb{R}^+$), $\mathbb{R}^n$, $\mathbb{R}^{n\times p}$ denote the sets of real (resp. real positive) scalars, n-dimensional vectors, $n\times p$ matrices, respectively. Let $\mathbf{I}_n$ and $\mathbf{1}_n$ be the $n\times n$ identity matrix, and n-dimensional column vector with all elements being one, respectively. Constant zero, zero vectors and zero matrices are all denoted by 0 if no conflict will arise. For a vector $x=(x_1,\dots,x_n)^T\in\mathbb{R}^n$, $\|x\|=\sqrt{x^Tx}$, $\|x\|_1=\sum_{i=1}^n|x_i|$, $\|x\|_p=(|x_1|^p+|x_2|^p+\cdots+|x_n|^p)^{1/p}$, and $\|x\|_\infty=\max_{i\in\{1,\dots,n\}}|x_i|$ denote the standard Euclidean norm, 1-norm, p-norm ($p\in[1,\infty]$), and ∞-norm, respectively. For a square matrix A, $tr(A)$ is its trace, $A>0$ (resp. $A\geq 0$) means that A is positive definite (resp. semi-definite) and $\lambda_2(A)$ is the minimum nonzero eigenvalue if $A\geq 0$. $\|B\|_F=\sqrt{tr(B^TB)}$ is the Frobenius norm of a matrix B which is not necessarily square. Let $\mathcal{I}_n=\{1,2,\dots,n\}$, and $\mathrm{col}(x_1,\dots,x_n)=({x_1}^T,\dots,{x_n}^T)^T$ is the column vectorization of vectors (matrices) x_i, $i\in\mathcal{I}_n$. $\mathrm{diag}(\cdot)$ is the diagonalization operator and $\mathrm{sgn}(\cdot)$ is the signum function defined component-wise.

An undirected (simple) graph $\mathcal{G}$ is specified by a node set $\mathcal{V}$ and an edge set $\mathcal{E}$. $(i,j)\in\mathcal{E}$ means that there are bidirectional links between i and j. In this case, j is a neighbor of i, denoted by $j\in\mathcal{N}_i$, and vice versa. The adjacency matrix of $\mathcal{G}$ is $\mathcal{A}=(a_{ij})\in\mathbb{R}^{|\mathcal{V}|\times|\mathcal{V}|}$, where $a_{ij}=1$ if $(i,j)\in\mathcal{E}$, and 0 otherwise. The incidence matrix of $\mathcal{G}$ (with a specific orientation) is $E=(E_{ik})\in\mathbb{R}^{|\mathcal{V}|\times|\mathcal{E}|}$, where $E_{ik}=1$ if the vertex i is the head of the edge k, -1 if i is the tail of k, and 0 otherwise. The Laplacian matrix of $\mathcal{G}$ is $\mathcal{L}=(l_{ij})\in\mathbb{R}^{|\mathcal{V}|\times|\mathcal{V}|}$, where $l_{ij}=-a_{ij}$, $i\neq j$, and $l_{ii}=\sum_{k=1,k\neq i}^{|\mathcal{V}|}a_{ik}$, $i\in\mathcal{I}_{|\mathcal{V}|}$. An undirected path on from vertex v_1 to v_s corresponds to a sequence of ordered edges (v_p,v_{p+1}), $p\in\mathcal{I}_{s-1}$. $\mathcal{G}$ is said to be connected if there exists a path between each pair of distinct nodes.

Lemma 1 ([6]): For a connected graph $\mathcal{G}$ with N nodes, $\mathcal{L}$ has a simple eigenvalue $\lambda_1=0$ with $\mathbf{1}_N$ as a corresponding right eigenvector, and the rest of the eigenvalues are positive. Moreover, $\lambda_2(\mathcal{L})=\min_{x\neq 0,\mathbf{1}_N^Tx=0}\frac{x^T\mathcal{L}x}{x^Tx}$.

Lemma 2 ([34]): For matrices A, B, C, D with appropriate dimensions, the following computation laws hold for the Kronecker product $\otimes$:

(1) $(\gamma A)\otimes B=A\otimes(\gamma B)=\gamma(A\otimes B)$, $\gamma\in\mathbb{R}$.
(2) $(A\otimes B)^T=A^T\otimes B^T$.
(3) $A\otimes(B+C)=A\otimes B+A\otimes C$.
(4) $(A\otimes B)(C\otimes D)=AC\otimes BD$.

Lemma 3 ([35]): (Hölder inequality) Let $(\mathcal{S},\Sigma,\mu)$ be a measure space and let $p,q\in[1,\infty]$ with $\frac{1}{p}+\frac{1}{q}=1$. Then, for all measurable functions f and g on $\mathcal{S}$, one has $\|fg\|\leq\|f\|_p\|g\|_q$.

B. Problem Statement

Consider a class of nonlinear MASs consisting of N agents as

$$\dot{x}_i(t) = Ax_i(t) + B(f_i(x_i) + u_i(t)), \quad i \in \mathcal{I}_N \tag{1}$$

where $x_i \in \mathbb{R}^n$, and $u_i \in \mathbb{R}^m$ are the state and control signals of agent i, respectively. A, B are known matrices with compatible dimensions such that (A, B) is stabilizable. $f_i(x_i) : \mathbb{R}^n \mapsto \mathbb{R}^m$ is an unknown but smooth nonlinear function modelling the matching inherent dynamics of agent i. Standard assumptions for the existence of unique solutions are made. The communication topology among the N agents is denoted by $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$.

A prescribed TVF can be described as an aggregated vector $h(t) = \mathrm{col}(h_1(t), h_2(t), \ldots, h_N(t))$, where $h_i(\cdot)$, $i \in \mathcal{I}_N$, is the piecewise continuously differential vector containing the formation command for agent i.

Definition 1: TVF $h(t)$ is said to be achieved by MASs (1) if, for any given bounded initial states, there holds $\lim_{t\to\infty}((x_i - h_i) - (x_j - h_j)) = 0, \forall i, j \in \mathcal{I}_N$.

The control goal is to design $u_i(t)$ for agent i, based on its available neighboring information, such that the TVF problem defined in the above can be tackled without the knowledge of the communication Laplacian spectrums.

Remark 1: The MAS dynamics considered in (1) is quite general, which covers first-order nonlinear one in [27], second-order nonlinear one in [28] and higher-order nonlinear one in [29], when (A, B) are chosen properly. Moreover, when $f_i(x_i) \equiv 0$, the problem reduces to the TVF for general linear time invariant MASs discussed in [13], [18], [19]. From Definition 1, the TVF problem degenerates to the specific distributed consensus problems [1], [33] when $h(t) \equiv 0$.

C. Approximation of $f_i(\cdot)$ Using Neural Networks

Neural networks are used to approximate the unknown nonlinearities of the system. Let $f_i(x_i)$ be smooth on a compact set $\Omega_x \subset \mathbb{R}^n$, then the well-known Stone-Weierstrass approximation theorem [36] guarantees that with a sufficient large scalar q, there exists an ideal weight matrix $W_i \in \mathbb{R}^{q\times m}$ such that

$$f_i(x_i) = W_i^T S_i(x_i) + \epsilon_i(t) \quad \forall x_i \in \Omega_x, \tag{2}$$

where $S_i(\cdot) : \mathbb{R}^m \mapsto \mathbb{R}^q$ is the stacked vector of activation functions of the q hidden layer neurons, and ϵ_i is the approximation error satisfying $\|\epsilon_i(t)\| \le \epsilon_{iM}$ with arbitrarily pre-designed error bound $\epsilon_{iM} > 0$.

The estimator of $f_i(x_i)$ is denoted by

$$\hat{f}_i(x_i) = \hat{W}_i^T(t) S_i(x_i), \tag{3}$$

where $\hat{W}_i(t)$ is a time-varying weight matrix to approximate W_i and will be designed later.

Denote $x = \mathrm{col}(x_1, \ldots, x_N)$, $f(x) = \mathrm{col}(f_1(x_1), \ldots, f_N(x_N))$, $W = \mathrm{diag}(W_1, \cdots, W_N)$, $S(x) = \mathrm{col}(S_1(x_1), \ldots, S_N(x_N))$, $\epsilon(t) = \mathrm{col}(\epsilon_1(t), \ldots, \epsilon_N(t))$, $\hat{f}(x) = \mathrm{col}(\hat{f}_1(x_1), \ldots, \hat{f}_N(x_N))$, $\hat{W}(t) = \mathrm{diag}(\hat{W}_1(t), \ldots, \hat{W}_N(t))$. Then, from (2) and (3), one has

$$f(x) = W^T S(x) + \epsilon(t), \tag{4}$$

$$\|\epsilon(t)\|_\infty \le \epsilon_M \tag{5}$$

for some threshold $\epsilon_M > 0$, and

$$\hat{f}(x) = \hat{W}^T(t) S(x). \tag{6}$$

The following gives some standard assumptions in the NN-based literature.

Assumption 1 ([27]–[29]):

- The unknown ideal NN weight matrix W_i is bounded $\forall i$, so that $\|W\|_F \le W_M$.
- The NN activation functions S_i are bounded $\forall i \in \mathcal{I}_N$ and $x \in \Omega_x$, so that $\|S(x)\|_F \le S_M$.

Remark 2: In Assumption 1, the constants W_M, S_M and ϵ_M could be unknown for the controller design, and serve only for the theoretical analysis. Besides, one can reach neither the ideal NN weight nor the zero functional approximation error in practice. Note that f is not allowed to depend explicitly on time, though this does not hurt the generality since one can easily turn the non-autonomous equations into autonomous ones by introducing $x_{n+1} = t$ [37].

III. Main Results

This section formulates two different strategies for TVF being node-based and edge-based, respectively. In order to clarify the design processes, some symbols are specified as $d_i = x_i - h_i$, $e_i = d_i - \frac{1}{N}\sum_{j=1}^N d_j$ and $\delta_i = \sum_{j=1}^N a_{ij}(d_i - d_j)$. Here d_i is the distance between current state and desired formation state of agent i, and e_i is the global formation error analysed from the viewpoint of agent i, while δ_i is the neighboring formation error actually measured by agent i. Moreover, let $h = \mathrm{col}(h_1, \ldots, h_N)$, $d = \mathrm{col}(d_1, \ldots, d_N)$, $e = \mathrm{col}(e_1, \ldots, e_N)$, $\delta = \mathrm{col}(\delta_1, \ldots, \delta_N)$ and $\Xi = \mathbf{I}_N - \frac{1}{N}\mathbf{1}_N\mathbf{1}_N^T$, then we have $e = (\Xi \otimes \mathbf{I}_n)d$ and $\delta = (\mathcal{L} \otimes \mathbf{I}_n)e$. It is obvious that Ξ has a simple eigenvalue 0 and $N-1$ multiple eigenvalue 1, and $\Xi^2 = \Xi$.

A. Fully Distributed TVF: Node-Based Strategy

In this subsection, fully-distributed TVF is analysed, where the control process does not depend on any global information.

Assumption 2: The communication topology $\mathcal{G}$ among the N agents is undirected and connected.

For agent i, let $\alpha_i(t)$ and $\beta_i(t)$ be some certain dynamic coupling strengths for neighboring feedback signal δ_i, and introduce $\overline{W}_i(t)$ as the pseudo ideal weight matrix for the NN estimator. Then, the dynamic controller for agent i, $i \in \mathcal{I}_N$, is proposed as

$$\begin{aligned} u_i &= K_0 x_i + K_1 d_i + \alpha_i(t) K_2 \delta_i \\ &\quad + \beta_i(t)\mathrm{sgn}(K_2\delta_i) - \hat{W}_i^T(t) S_i(x_i) \\ \dot{\alpha}_i &= \rho_i \delta_i^T \Gamma \delta_i \\ \dot{\beta}_i &= \nu_i \|K_2\delta_i\|_1 \\ \dot{\hat{W}}_i &= \tau_i[S_i(x_i)\delta_i^T P^{-1} B - \sigma(\hat{W}_i - \overline{W}_i(t))] \\ \dot{\overline{W}}_i &= \psi_i \sigma(\hat{W}_i - \overline{W}_i), \end{aligned} \tag{7}$$

Algorithm 1: Fully-distributed TVF Controller Design

1) Find K_0 such that the following TVF feasibility condition
$$(A+BK_0)(h_i-h_j)-(\dot{h}_i-\dot{h}_j)=0 \tag{8}$$
holds $\forall i\in\mathcal{I}_N$ and $j\in\mathcal{N}_i$. If such K_0 exists, continue; else, the algorithm stops, in which case the specified TVF $h(t)$ is not feasible for MAS (1) under protocol (7).
2) Choose K_1 such that $(A+BK_0+BK_1,B)$ is stabilizable and η, $\theta\in\mathbb{R}^+$, then solve the following Linear Matrix Inequality (LMI):
$$\begin{aligned}(A+BK_0+BK_1)P+P(A+BK_0+BK_1)^T&\\ -\eta BB^T+\theta P\leq 0&\end{aligned} \tag{9}$$
to get a $P>0$.
3) Design $K_2=-B^TP^{-1}$, $\Gamma=P^{-1}BB^TP^{-1}$ and choose small scalars $\rho_i,\nu_i,\tau_i,\sigma,\psi_i$.

where K_0, K_1, K_2 and Γ are feedback gain matrices, $\rho_i,\nu_i,\tau_i,\sigma,\psi_i\in\mathbb{R}^+$, and $P>0$ is a positive definite matrix. These parameters will be designed later.

Remark 3: The controller consists of three feedback terms and a compensation term. The gain K_0 is to be designed to make the time-varying formation $h(\cdot)$ feasible; the gain K_1 controls the average formation signal, i.e., $\frac{1}{N}\sum_{j\in\mathcal{I}_N}d_j$; finally, the gain K_2 is a consensus gain. The compensation term is to eliminate after identifying the nonlinear effects online.

The whole design process is summarized in Algorithm 1, and analyzed in the following theorem.

Theorem 1: Under Assumptions 1 and 2, and feasibility condition (8), the TVF problem in Definition 1 can be solved with protocol (7) along with the parameters designed in Algorithm 1.

Proof: Denote $\tilde{W}_i(t)=\hat{W}_i(t)-W_i$ as the NN weight approximation errors. Substitute $f_i(x_i)$ in (2) and u_i in (7) into (1), and some manipulation gives the dynamics of x_i as
$$\begin{aligned}\dot{x}_i=&(A+BK_0+BK_1)x_i+\alpha_iBK_2\delta_i+\beta_iB\mathrm{sgn}(K_2\delta_i)\\&-B(\tilde{W}_i^TS_i(x_i)-\epsilon_i)-BK_1h_i.\end{aligned} \tag{10}$$
Let $\mathcal{M}_1(t)=\mathrm{diag}(\alpha_1,\ldots,\alpha_N)$, $\mathcal{M}_2(t)=\mathrm{diag}(\beta_1,\ldots,\beta_N)$ and $\tilde{W}(t)=\hat{W}(t)-W$, one has the dynamics of the stacked states x as
$$\begin{aligned}\dot{x}=&[\mathbf{I}_N\otimes(A+BK_0+BK_1)]x+(\mathcal{M}_1\otimes BK_2)\delta\\&+(\mathcal{M}_2\otimes B)\mathrm{sgn}((\mathbf{I}_N\otimes K_2)\delta)\\&-(\mathbf{I}_N\otimes B)(\tilde{W}^TS(x)-\epsilon)-(\mathbf{I}_N\otimes BK_1)h.\end{aligned} \tag{11}$$
Thereafter, the dynamics of d and e can be obtained as
$$\begin{aligned}\dot{d}=&[\mathbf{I}_N\otimes(A+BK_0+BK_1)]d+(\mathcal{M}_1\mathcal{L}\otimes BK_2)d\\&+(\mathcal{M}_2\otimes B)\mathrm{sgn}((\mathcal{L}\otimes K_2)d)\\&-(\mathbf{I}_N\otimes B)(\tilde{W}^TS(x)-\epsilon)\\&+(\mathbf{I}_N\otimes(A+BK_0))h-\dot{h},\end{aligned} \tag{12}$$
$$\begin{aligned}\dot{e}=&[\mathbf{I}_N\otimes(A+BK_0+BK_1)+\varXi\mathcal{M}_1\mathcal{L}\otimes BK_2]e\\&+(\varXi\mathcal{M}_2\otimes B)\mathrm{sgn}((\mathcal{L}\otimes K_2)e)\\&-(\varXi\otimes B)(\tilde{W}^TS(x)-\epsilon)\\&+(\varXi\otimes(A+BK_0))h-(\varXi\otimes\mathbf{I}_n)\dot{h},\end{aligned} \tag{13}$$
where we have used Lemma 2 and the fact that, under Assumption 2, $\mathcal{L}\varXi=\varXi\mathcal{L}=\mathcal{L}$. It is clear that $e=0$ is equivalent to $d_1=d_2=\cdots=d_N$, i.e., the TVF is realized.

Denote $\tau=\mathrm{diag}(\tau_1,\ldots,\tau_N)$, $\tilde{\overline{W}}=\overline{W}-W$, $\psi=\mathrm{diag}(\psi_1,\ldots,\psi_N)$. Consider the Lyapunov function candidate as
$$\begin{aligned}V_1=&\frac{1}{2}e^T(\mathcal{L}\otimes P^{-1})e\\&+\frac{1}{2}\sum_{i=1}^{N}[\frac{(\alpha_i(t)-\bar{\alpha})^2}{\rho_i}+\frac{(\beta_i(t)-\bar{\beta})^2}{\nu_i}]\\&+\frac{1}{2}tr(\tilde{W}^T\tau^{-1}\tilde{W}+\tilde{\overline{W}}^T\psi^{-1}\tilde{\overline{W}})\end{aligned} \tag{14}$$
where P is a solution of (9) and $\bar{\alpha},\bar{\beta}\in\mathbb{R}^+$ are to be determined later.

Given (7), the time derivative of V_1 along (13) is
$$\begin{aligned}\dot{V}_1=&e^T[\mathcal{L}\otimes P^{-1}(A+BK_0+BK_1)+\mathcal{L}\mathcal{M}_1\mathcal{L}\otimes P^{-1}BK_2]e\\&+e^T(\mathcal{L}\mathcal{M}_2\otimes P^{-1}B)\mathrm{sgn}((\mathcal{L}\otimes K_2)e)\\&-e^T(\mathcal{L}\otimes P^{-1}B)(\tilde{W}^TS(x)-\epsilon)\\&+e^T(\mathcal{L}\otimes P^{-1}(A+BK_0))h-e^T(\mathcal{L}\otimes P^{-1})\dot{h}\\&+\sum_{i=1}^{N}(\alpha_i(t)-\bar{\alpha})\delta_i^T\Gamma\delta_i+\sum_{i=1}^{N}(\beta_i(t)-\bar{\beta})\|K_2\delta_i\|_1\\&+\sum_{i=1}^{N}tr(\tilde{W}_i^T(S_i(x_i)\delta_i^TP^{-1}B-\sigma(\hat{W}_i-\overline{W}_i)))\\&+\sigma\sum_{i=1}^{N}tr(\tilde{\overline{W}}_i^T(\hat{W}_i-\overline{W}_i)).\end{aligned} \tag{15}$$
Now that the feasibility condition (8) is satisfied, one has
$$(\mathcal{L}\otimes(A+BK_0))h-(\mathcal{L}\otimes\mathbf{I}_n)\dot{h}=0. \tag{16}$$
Then, according to step 3) of Algorithm 1, one has
$$\begin{aligned}&e^T(\mathcal{L}\mathcal{M}_1\mathcal{L}\otimes P^{-1}BK_2)e\\&=-e^T(\mathcal{L}\otimes\mathbf{I}_n)(\mathcal{M}_1\otimes P^{-1}BB^TP^{-1})(\mathcal{L}\otimes\mathbf{I}_n)e\\&=-\delta^T(\mathcal{M}_1\otimes\Gamma)\delta\\&=-\sum_{i=1}^{N}\alpha_i(t)\delta_i^T\Gamma\delta_i\end{aligned} \tag{17}$$
and
$$\begin{aligned}&e^T(\mathcal{L}\mathcal{M}_2\otimes P^{-1}B)\mathrm{sgn}((\mathcal{L}\otimes K_2)e)\\&=-\delta^T(\mathcal{M}_2\otimes K_2^T)\mathrm{sgn}((\mathbf{I}_N\otimes K_2)\delta)\\&=-\sum_{i=1}^{N}\beta_i(t)\delta_i^TK_2^T\mathrm{sgn}(K_2\delta_i)\\&=-\sum_{i=1}^{N}\beta_i(t)\|K_2\delta_i\|_1,\end{aligned} \tag{18}$$
where we have used the fact that $x^T\mathrm{sgn}(x)=\|x\|_1$ for an arbitrary real column vector x to get the last equality.

Moreover, since $tr(XY)=tr(YX)$ holds for any compatible matrices X,Y, one has

$$e^T(\mathcal{L}\otimes P^{-1}B)(\tilde{W}^T S(x)) = \sum_{i=1}^{N} tr(\tilde{W}_i^T S_i(x_i)\delta_i^T P^{-1}B). \quad (19)$$

Combining (15)-(19) and some manipulations gives

$$\begin{aligned}\dot{V}_1 &= e^T[\mathcal{L}\otimes P^{-1}(A+BK_0+BK_1)]e \\ &\quad + e^T(\mathcal{L}\otimes P^{-1}B)\epsilon \\ &\quad - \bar{\alpha}\sum_{i=1}^{N}\delta_i^T\Gamma\delta_i - \bar{\beta}\sum_{i=1}^{N}\|K_2\delta_i\|_1 \\ &\quad - \sigma\|\hat{W}-\overline{W}\|_F^2. \end{aligned} \quad (20)$$

Introduce $\bar{e} = (\mathbf{I}_N\otimes P^{-1})e$, one can obtain that

$$\begin{aligned}\dot{V}_1 &= \frac{1}{2}\bar{e}^T[\mathcal{L}\otimes((A+BK_0+BK_1)P \\ &\qquad + P(A+BK_0+BK_1)^T) - 2\bar{\alpha}\mathcal{L}^2\otimes BB^T]\bar{e} \\ &\quad + \bar{e}^T(\mathcal{L}\otimes B)\epsilon - \bar{\beta}\|(\mathcal{L}\otimes B^T)\bar{e}\|_1 \\ &\quad - \sigma\|\hat{W}-\overline{W}\|_F^2 \\ &\le \frac{1}{2}\bar{e}^T[\mathcal{L}\otimes((A+BK_0+BK_1)P \\ &\qquad + P(A+BK_0+BK_1)^T) - 2\bar{\alpha}\mathcal{L}^2\otimes BB^T]\bar{e} \\ &\quad - (\bar{\beta}-\epsilon_M)\|(\mathcal{L}\otimes B^T)\bar{e}\|_1, \end{aligned} \quad (21)$$

where (5) and Lemma 3 have been used to get the inequality.

Since $\mathcal{G}$ is connected, it follows from Lemma 1 that $\mathcal{L}$ is positive semi-definite with a simple eigenvalue 0. Then there exists a unitary matrix $U = [\frac{\mathbf{1}_N}{\sqrt{N}}, Y_1]$ with $Y_1 \in \mathbb{R}^{N\times(N-1)}$, such that $U^T\mathcal{L}U = \Lambda \triangleq \mathrm{diag}(0,\lambda_2,\ldots,\lambda_N)$, where $\lambda_2\le\cdots\le\lambda_N$ are the positive eigenvalues of $\mathcal{L}$. By further letting $\hat{e} = \mathrm{col}(\hat{e}_1,\ldots,\hat{e}_N) = (U^T\otimes\mathbf{I}_n)\bar{e}$, one has $\hat{e}_1 = (\frac{\mathbf{1}_N^T}{\sqrt{N}}\otimes\mathbf{I}_n)\bar{e} = 0$ immediately. Then, it can be derived from (21) that

$$\begin{aligned}\dot{V}_1 &\le \frac{1}{2}\bar{e}^T[\mathcal{L}\otimes((A+BK_0+BK_1)P \\ &\qquad + P(A+BK_0+BK_1)^T) - 2\bar{\alpha}\mathcal{L}^2\otimes BB^T]\bar{e} \\ &\quad - (\bar{\beta}-\epsilon_M)\|(\mathcal{L}\otimes B^T)\bar{e}\|_1 \\ &= \frac{1}{2}\sum_{i=2}^{N}\lambda_i\hat{e}_i^T((A+BK_0+BK_1)P \\ &\qquad + P(A+BK_0+BK_1)^T - 2\bar{\alpha}\lambda_i BB^T)\hat{e}_i \\ &\quad - (\bar{\beta}-\epsilon_M)\|(\mathcal{L}\otimes B^T)\bar{e}\|_1. \end{aligned} \quad (22)$$

Choose $\bar{\alpha}$ and $\bar{\beta}$ sufficiently large such that $\bar{\alpha} > \frac{\eta}{2\lambda_2(\mathcal{L})}$ and $\bar{\beta} > \epsilon_M$, then it follows from (22) and LMI (9) that

$$\dot{V}_1 \le -\frac{\theta}{2}\hat{e}^T(\Lambda\otimes P)\hat{e} = -\frac{\theta}{2}e^T(\mathcal{L}\otimes P^{-1})e. \quad (23)$$

Now $\mathcal{L}\ge 0$ and $P > 0$, then $V_1(t)$ is nonincreasing, which guarantees that all the signals e, $\alpha_i(t)$, $\beta_i(t)$, $\tilde{W}$ and $\overline{W}$ in $V_1(t)$ are bounded. From (13) and Assumption 1, $\dot{e}$ is also bounded, then the function $\frac{\theta}{2}e^T(\mathcal{L}\otimes P^{-1})e$ is uniformly continuous. Since $V_1(t)\le V_1(0)$ and is nonincreasing, it thus has a finite limit V_1^∞ as $t\to\infty$. In fact,

$$\int_0^{+\infty}\frac{\theta}{2}e(t)^T(\mathcal{L}\otimes P^{-1})e(t)dt \le \frac{V_1(0)-V_1^\infty}{2}. \quad (24)$$

By utilizing the well-known Barbalat's Lemma [38], $\lim_{t\to\infty}\frac{\theta}{2}e^T(\mathcal{L}\otimes P^{-1})e = 0$. Note that $\hat{e}_1 = 0$, one has $\lim_{t\to\infty}e = 0$, meaning that $\lim_{t\to\infty}((x_i-h_i)-(x_j-h_j)) = 0,\ \forall i,j\in\mathcal{I}_N$. The TVF is realized and the proof is ended. ■

Remark 4: The feasibility condition (8) (as well as (26) in subsection III-B) is necessary for TVF problems, and can also be found in [13], [18]. From (8), the gain K_0 is to make the TVF $h(\cdot)$ feasible as stated in Remark 3. Traditional methods for time-invariant formations [39] can not be used directly for TVF problems in this paper.

Remark 5: In fact, the existence of the signum function in (10) reflects that the dynamics of x, and consequently of e, are both discontinuous. Therefore, the stability analysis should be performed in context of differential inclusions and nonsmooth theory [40]. Since the signum function is measurable and essentially bounded, the solutions for (13) always exist in the sense of Filippov. Note that the Lyapunov candidate (14) is continuous differentiable and its set-valued Lie derivative is a singleton at the discontinuous point, the proof still holds. More details can be found in [33] and the references therein. Along this paper, differential inclusions are not incorporated in the proofs to avoid symbol redundancy.

B. TVF on Switching Graphs: Edge-Based Strategy

In this subsection, a class of edge-based protocol is studied, which is found to be applicable for MASs with switching connected communications. This part is the natural extension from consensus control in precious work to formation control of MASs.

The communication topology of the agents at time t is denoted by $\mathcal{G}(t)\in\mathcal{G}^s$, where $\mathcal{G}^s = \{\mathcal{G}^1,\mathcal{G}^2,\ldots,\mathcal{G}^l\}$ is the set of all possible communication graphs. The corresponding incidence matrix is denoted by the time-varying matrix $E(t)$, and $j\in\mathcal{N}_i^t$ if agent j is a neighbor of agent i at time t.

Assumption 3: All the l possible communication graphs in $\mathcal{G}^s$ are undirected and connected.

For each possible link between agent i and j, let α_{ij} and β_{ij} be dome certain dynamic coupling strength for the feedback signal $d_i - d_j$, and consistently let $\overline{W}_i(t)$ be the pseudo ideal weight matrix for the NN estimator. Then, the dynamic controller for agent i, $i\in\mathcal{I}_N$, is proposed as

$$\begin{aligned} u_i &= K_0x_i + K_1d_i + \sum_{j\in\mathcal{N}_i^t}\alpha_{ij}(t)K_2(d_i-d_j) \\ &\qquad + \sum_{j\in\mathcal{N}_i^t}\beta_{ij}(t)\mathrm{sgn}(K_2(d_i-d_j)) \\ &\qquad - \hat{W}_i^T(t)S_i(x_i) \\ \dot{\alpha}_{ij} &= \rho_{ij}(d_i-d_j)^T\Gamma(d_i-d_j) \quad j\in\mathcal{N}_i^t \\ \dot{\beta}_{ij} &= \nu_{ij}\|K_2(d_i-d_j)\|_1 \qquad j\in\mathcal{N}_i^t \\ \dot{\hat{W}}_i &= \tau_i[S_i(x_i)e_i^TP^{-1}B - \sigma(\hat{W}_i-\overline{W}_i(t))] \\ \dot{\overline{W}}_i &= \psi_i\sigma(\hat{W}_i-\overline{W}_i), \end{aligned} \quad (25)$$

Algorithm 2: Controller Design for TVF on Switching Graphs

1) Find K_0 such that the following TVF feasibility condition

$$(A+BK_0)h_i-\dot{h}_i=0 \tag{26}$$

holds $\forall i\in\mathcal{I}_N$. If such K_0 exists, continue; else, the algorithm stops, in which case the specified TVF $h(t)$ is not feasible for MAS (1) under protocol (25).

2) Design K_1, K_2 and Γ following step 2) and step 3) successively in Algorithm 1, and choose small scalars $\rho_{ij},\nu_{ij},\tau_i,\sigma,\psi_i$.

where K_0, K_1, K_2 and Γ are feedback gain matrices, $\rho_{ij},\nu_{ij},\tau_i,\sigma,\psi_i\in\mathbb{R}^+$, and $P>0$ is a positive definite matrix.

Initialize $\alpha_{ij}(0)=\alpha_{ji}(0)$ and $\beta_{ij}(0)=\beta_{ji}(0)$, then both α_{ij} and β_{ij} admit certain symmetric property, for instance, $\alpha_{ij}(t)=\alpha_{ji}(t)$.

Similarly, the design process of this protocol is summarized in Algorithm 2, and analyzed in the following theorem.

Theorem 2: Under Assumptions 1 and 3, and feasibility condition (26), the TVF problem in Definition 1 can be solved with protocol (25) along with parameters designed in Algorithm 2.

Proof: Without specific instructions, the symbols of the NN estimators in this proof are the same as those in the proof of Theorem 1. Substitute $f_i(x_i)$ in (2) and u_i in (25) into (1), and notice the definition of d_i, one can compute the dynamics of d as

$$\begin{aligned}\dot{d}&=[\mathbf{I}_N\otimes(A+BK_0+BK_1)]d+(E\mathcal{M}_3E^T\otimes BK_2)d\\&\quad+(E\mathcal{M}_4\otimes B)\mathrm{sgn}((E^T\otimes K_2)d)\\&\quad-(\mathbf{I}_N\otimes B)(\tilde{W}^TS(x)-\epsilon)\\&\quad+(\mathbf{I}_N\otimes(A+BK_0))h-\dot{h},\end{aligned}\tag{27}$$

where $\mathcal{M}_3(t)=\mathrm{diag}(\alpha_{ij})$, $\mathcal{M}_4(t)=\mathrm{diag}(\beta_{ij})$. Here both the components α_{ij}, β_{ij} are fed in congruent with the edge index of the incidence matrix $E(t)$, i.e., the column index.

Since the feasibility condition (26) is satisfied, one has $(\mathbf{I}_N\otimes(A+BK_0))h-\dot{h}=0$. Under Assumption 3, $\varXi E(t)=E(t)$ holds $\forall t$. Moreover, it is known that $e=(\varXi\otimes\mathbf{I}_n)d$, so one has the dynamics of e as

$$\begin{aligned}\dot{e}&=[\mathbf{I}_N\otimes(A+BK_0+BK_1)+E\mathcal{M}_3E^T\otimes BK_2]e\\&\quad+(E\mathcal{M}_4\otimes B)\mathrm{sgn}((E^T\otimes K_2)e)\\&\quad-(\varXi\otimes B)(\tilde{W}^TS(x)-\epsilon)\end{aligned}\tag{28}$$

Consider the common Lyapunov function candidate:

$$\begin{aligned}V_2&=\frac{1}{2}e^T(\varXi\otimes P^{-1})e\\&\quad+\frac{1}{4}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i^t}\left[\frac{(\alpha_{ij}(t)-\tilde{\alpha})^2}{\rho_{ij}}+\frac{(\beta_{ij}(t)-\tilde{\beta})^2}{\nu_{ij}}\right]\\&\quad+\frac{1}{2}tr(\tilde{W}^T\tau^{-1}\tilde{W}+\tilde{\overline{W}}^T\psi^{-1}\tilde{\overline{W}}),\end{aligned}\tag{29}$$

where P is a solution of (9) and $\tilde{\alpha},\tilde{\beta}\in\mathbb{R}^+$ are to be determined later.

The derivative of V_2 along (28) under (25) can be obtained as

$$\begin{aligned}\dot{V}_2&=e^T[\varXi\otimes P^{-1}(A+BK_0+BK_1)\\&\qquad+E\mathcal{M}_3E^T\otimes P^{-1}BK_2]e\\&\quad+e^T(E\mathcal{M}_4\otimes P^{-1}B)\mathrm{sgn}((E^T\otimes K_2)e)\\&\quad-e^T(\varXi\otimes P^{-1}B)(\tilde{W}^TS(x)-\epsilon)\\&\quad+\frac{1}{2}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i^t}[(\alpha_{ij}(t)-\tilde{\alpha})(d_i-d_j)^T\Gamma(d_i-d_j)\\&\qquad+(\beta_{ij}(t)-\tilde{\beta})\|K_2(d_i-d_j)\|_1]\\&\quad+\sum_{i=1}^N tr(\tilde{W}_i^T(S_i(x_i)e_i^TP^{-1}B-\sigma(\hat{W}_i-\overline{W}_i)))\\&\quad+\sigma\sum_{i=1}^N tr(\tilde{\overline{W}}_i^T(\hat{W}_i-\overline{W}_i)).\end{aligned}\tag{30}$$

Since $K_2=-B^TP^{-1}$ and $\Gamma=P^{-1}BB^TP^{-1}$, one has

$$\begin{aligned}&e^T(E\mathcal{M}_3E^T\otimes P^{-1}BK_2)e\\&=-\frac{1}{2}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i^t}\alpha_{ij}(t)(e_i-e_j)^TP^{-1}BB^TP^{-1}(e_i-e_j)\\&=-\frac{1}{2}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i^t}\alpha_{ij}(t)(d_i-d_j)^T\Gamma(d_i-d_j),\end{aligned}\tag{31}$$

and

$$\begin{aligned}&e^T(E\mathcal{M}_4\otimes P^{-1}B)\mathrm{sgn}((E^T\otimes B^TP^{-1})e)\\&\quad=\frac{1}{2}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i^t}\beta_{ij}(t)\|B^TP^{-1}(e_i-e_j)\|_1\\&\quad=-\frac{1}{2}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i^t}\beta_{ij}(t)\|K_2(d_i-d_j)\|_1.\end{aligned}\tag{32}$$

Furthermore, similar to (19), one has

$$e^T(\varXi\otimes P^{-1}B)(\tilde{W}^TS(x))=\sum_{i=1}^N tr(\tilde{W}_i^TS_i(x_i)e_i^TP^{-1}B).\tag{33}$$

It then follows from (30)-(33) and some manipulations that

$$\begin{aligned}\dot{V}_2&=e^T[\varXi\otimes P^{-1}(A+BK_0+BK_1)]e\\&\quad+e^T(\varXi\otimes P^{-1}B)\epsilon\\&\quad-\frac{1}{2}\sum_{i=1}^N\sum_{j\in\mathcal{N}_i^t}\tilde{\alpha}(d_i-d_j)^T\Gamma(d_i-d_j)\\&\qquad+\tilde{\beta}\|K_2(d_i-d_j)\|_1]\\&\quad-\sigma\|\hat{W}-\overline{W}\|_F^2.\end{aligned}\tag{34}$$

Let $\bar{e} = (\mathbf{I}_N \otimes P^{-1})e$ and note $\mathcal{L}(t) = EE^T$ one has

$$\begin{aligned}\dot{V}_2 = &\frac{1}{2}\bar{e}^T[\Xi \otimes ((A + BK_0 + BK_1)P \\ &\quad + P(A + BK_0 + BK_1)^T) - 2\tilde{\alpha}\mathcal{L}(t) \otimes BB^T]\bar{e} \\ &+ \bar{e}^T(\Xi \otimes B)\epsilon - \bar{\beta}\|(\Xi \otimes B^T)\bar{e}\|_1 \\ &- \sigma\|\hat{W} - \overline{W}\|_F^2 \\ \leq &\frac{1}{2}\bar{e}^T[\Xi \otimes ((A + BK_0 + BK_1)P \\ &+ P(A + BK_0 + BK_1)^T) - 2\tilde{\alpha}\mathcal{L}(t) \otimes BB^T]\bar{e} \\ &- (\bar{\beta} - \epsilon_M)\|(\Xi \otimes B^T)\bar{e}\|_1. \end{aligned} \tag{35}$$

Denote $\underline{\lambda} = \min\{\lambda_2(\mathcal{L}^1), \lambda_2(\mathcal{L}^2), \ldots, \lambda_2(\mathcal{L}^l)\}$ and choose $\tilde{\alpha}$, $\tilde{\beta}$ such that $\tilde{\alpha} > \frac{\eta}{2\underline{\lambda}}$ and $\tilde{\beta} > \epsilon_M$. Then it follows from (35) and LMI (9) that

$$V_2 \leq -\frac{\theta}{2}\bar{e}^T(\Xi \otimes P)\bar{e} = -\frac{\theta}{2}e^T(\Xi \otimes P^{-1})e. \tag{36}$$

The following analysis is similar as that for (23) in the proof of Theorem (1), and the Barbalat's Lemma guarantees that $\lim_{t\to\infty} e = 0$, i.e., the TVF is realized. The proof is completed. ∎

Remark 6: It should be mentioned that the proposed controllers (7) and (25) are also valid when the dynamics of x_i suffers from matching bounded disturbance ω_i, as is considered in [30], [31]. This can be shown by a few modifications in the proofs, thus the details are omitted for brevity.

Remark 7: Since the NN approximation processes are valid on a compact set $\Omega_x \subset \mathbb{R}^n$, our results are semi-global once the structure of the NNs is selected. However, if $f_i(x_i)$ is smooth over $\mathbb{R}^n$ and as many hidden neurons can be chosen, the size of Ω_x can be arbitrarily large in the NNs. In this case, the results will be global [41]. Nevertheless, in comparison with tradition NN-based methods in [27], [29], [30] where the residual errors are SGUUB, SGA-TVF is realized in this paper.

Remark 8: A practical issue when implementing the protocols is that the dynamic coupling gains may increase slowly because of numerical errors, chattering effects or disturbances [42], and even destroy the stability as $t \to \infty$. To handle this issue, one can introduce some small scalar r, then update α_i, β_i in (7) whenever $\|\delta_i\| > r$, and α_{ij}, β_{ij} in (25) whenever $\|d_i - d_j\| > r$. On the contrary of either case, one can hold steadily the corresponding coupling gains. Then, as long as the TVF errors converge into some desirable bound, the adaptive parameters converge to some finite values.

Let us take a close look at the protocols and the proofs. On the one hand, the node-based design (7) is fully-distributed, but only applicable to MASs with fixed communication topology since the Lyapunov candidate (14) is dependent on the Laplacian matrix $\mathcal{L}$. On the other hand, though the edge-based design (25) does need a central data collector recoding d_i, $i \in \mathcal{I}_N$, when estimating the ideal NN weight matrices, it can be applicable to MASs with switching communication graphs. These interesting observations help us decide that which protocol is preferential in practice, as will be seen in the following numerical examples.

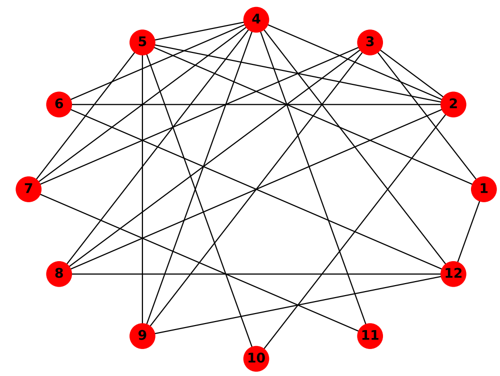

Fig. 1. Communication topology for Example 1.

IV. Numerical Examples

In this section, two numerical examples are presented to verify the theoretical results. For both examples, single layer NNs with 36 hidden neurons and sigmoid hidden activation functions are utilized to approximate the unknown nonlinear functions. The initial positions of the agents are chosen from a Gaussian distribution with standard deviation 3.

A. Fully-Distributed TVF: Node-Based

It is common in practical network systems that the communication channels are fixed but their quantitativeness is relatively large. In this case, node-based protocol (7) is superior to solve the TVF problem.

Example 1: Consider a third-order MAS (1) with $N = 12$,

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \\ -2 & -10 & -3 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

$$f_i(x_i(t)) = ix_{i1}\sin(x_{i2}), \quad w_i(t) = 0.1\cos(it), \tag{37}$$

where $w_i(t)$ is the matching disturbance in Remark 6. The communication topology is chosen as in Fig. 1. The required TVF is two nested hexagons with $h_i(t) = (\sin(t + \frac{(i-1)\pi}{3}), -\cos(t + \frac{(i-1)\pi}{3}), 2\cos(t + \frac{(i-1)\pi}{3})^T$ for $i \in \mathcal{I}_6$, and $h_i(t) = \frac{1}{2}h_{i-6}(t)$ for $i \in \mathcal{I}_{12} - \mathcal{I}_6$.

Let $K_0 = (0, 4, 0)$, it is easy to verify that the feasibility condition (8) holds. Since $A + BK_0$ is stabilizable, we choose $K_1 = (0, 0, 0)$. Following Algorithm 1, let $\eta = 2$ and $\theta = 1$, and solve (9) to give

$$P = \begin{pmatrix} 5.9910 & 0.1493 & -5.9611 \\ 0.1493 & 0.3866 & -1.1662 \\ -5.9611 & -1.1662 & 9.1486 \end{pmatrix}.$$

Set

$$K_2 = (-1.8121 - 5.1855 - 1.9510),$$

$$\Gamma = \begin{pmatrix} 3.2836 & 9.3966 & 3.5354 \\ 9.3966 & 26.8899 & 10.1172 \\ 3.5354 & 10.1172 & 3.8065 \end{pmatrix},$$

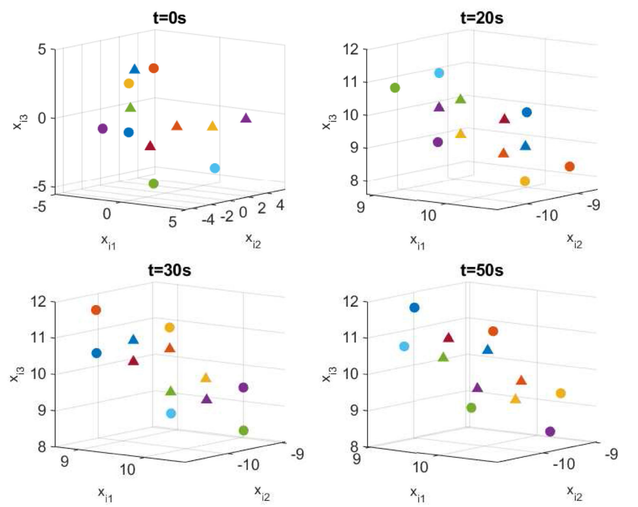

Fig. 2. Snapshots of all agents at $t = 0\ s, 20\ s, 30\ s$, and $50\ s$. The circles and triangles are used to mark the agents $i \in \mathcal{I}_6$ and the agents $i \in \mathcal{I}_{12} - \mathcal{I}_6$, respectively.

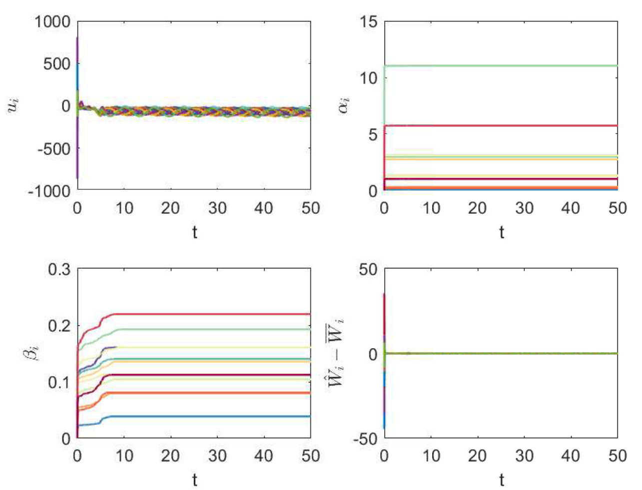

Fig. 3. Control inputs u_i (upper left), adaptive coupling weights α_i (upper right) and β_i (lower left), and component-wise values of $\hat{W}_i - \overline{W}_i$ (lower right), $i = 1, \ldots, 6$.

and choose $\rho_i = \nu_i = 0.1$, $\tau_i = \psi_i = 100$, $\sigma = 0.6$. Then all the conditions of Theorem 1 are satisfied. Considering Remark 8, one can introduce $r = 0.1$.

Solve the TVF with protocol (7), and several snapshots of the agents are provided in Fig. 2, showing that the nested hexagons emerge and keep rotating synchronously. The control inputs as well as the auxiliary variables are shown in Fig. 3. It can be seen that all the adaptive coupling weights α_i and β_i converge to some finite values, and the estimated ideal weight matrices converge to the pseudo ones.

Define the global formation error as $e_f(t) = \sqrt{\frac{1}{12}\sum_{i=1}^{12} \|e_i(t)\|^2}$, then e_f converges to zero, as shown in Fig. 4. To highlight the necessity of the NN compensation term in the proposed protocols, we drop off the last term of u_i in (7). The global formation error does not converge to zero, and only practical TVF with bounded formation error is realized, as shown in the right half of Fig. 4.

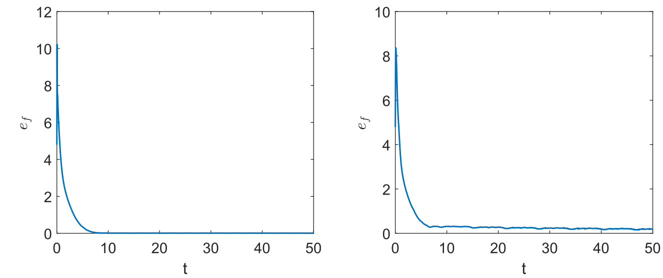

Fig. 4. The global formation error with (left) and without (right) NN compensation term in (7).

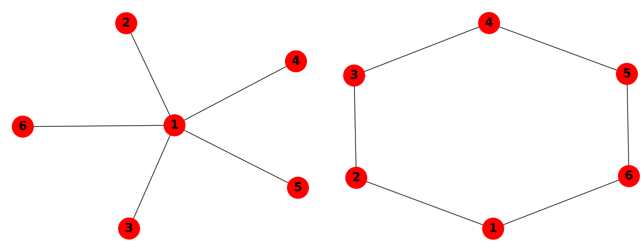

Fig. 5. The possible communication topologies (Star and Ring) for Example 2.

B. TVF on Switching Graphs: Edge-Based

In many cases, the communication graphs in the network systems are much sparser, but may switch within several modes. In these situations, the edge-based protocol (25) is an effective candidate for the TVF on switching graphs.

Example 2: Consider a second-order MAS (1) with $N = 6$,

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$
$$f_i(x_i(t)) = i x_{i1} x_{i2}, \quad w_i(t) = 0.1\cos(it), \tag{38}$$

The communication graph is assumed randomly switching every 2 seconds between the star graph and the ring graph depicted in Fig. 5. The desired TVF is an equilateral triangle specified by $h_i(t) = (2\sin(t + \frac{2(i-1)\pi}{3}), 2\cos(t + \frac{2(i-1)\pi}{3}))^T$ for $i \in \mathcal{I}_3$, and $h_i(t) = (\sin(t + \frac{2(i-1)\pi}{3} + \pi), \cos(t + \frac{2(i-1)\pi}{3} + \pi))^T$ for $i \in \mathcal{I}_6 - \mathcal{I}_3$.

With $K_0 = (0, -2)$, the feasibility condition (26) hold. Following Algorithm 1, choose $K_1 = K_0$, $\eta = 3$ and $\theta = 2$, and solve (9) to give

$$P = \begin{pmatrix} 0.6855 & -0.7725 \\ -0.7725 & 1.2465 \end{pmatrix}.$$

Set

$$K_2 = (-2.9967, -2.6594),$$

$$\Gamma = \begin{pmatrix} 8.9805 & 7.9694 \\ 7.9694 & 7.0722 \end{pmatrix},$$

Let $\rho_{ij} = \nu_{ij} = 0.02$ and again $\tau_i = \psi_i = 100$, $\sigma = 0.5$, all the conditions of Theorem 2 are satisfied. Considering Remark 8, we finally choose $r = 0.05$.

The switching signal is shown in Fig. 6. The profiles of the agents are in Fig. 7, showing how the triangle emerges and

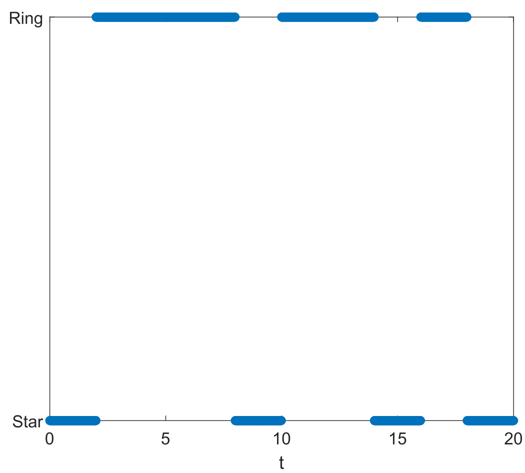

Fig. 6. Switching signal.

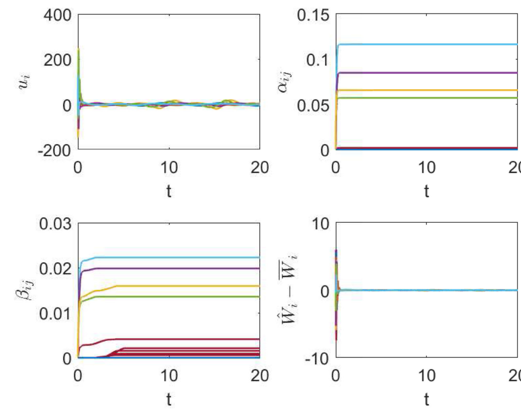

Fig. 8. Control inputs u_i (upper left), adaptive coupling weights α_{ij} (upper right) and β_{ij} (lower left), and component-wise values of $\hat{W}_i - \overline{W}_i$ (lower right), $i \in \mathcal{I}_6$, $j \in \mathcal{N}_i^t$.

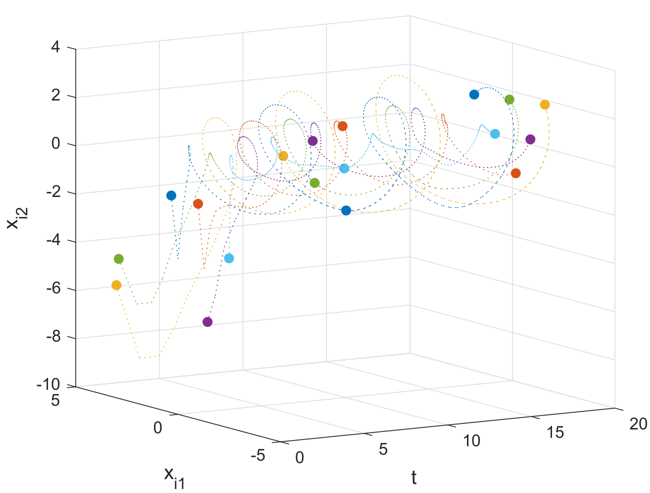

Fig. 7. Profiles of the agents x_i, $i \in \mathcal{I}_6$, where the broken lines are used to denote the trajectories of the agents, and the circles are used to mark the agents at $t = 0\ s, 10\ s$ and $20\ s$.

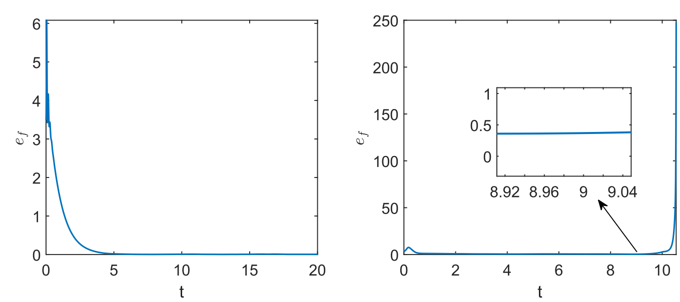

Fig. 9. The global formation error with (left) and without (right) NN compensation term in (25).

rotates. The control inputs as well as the auxiliary variables are provided in Fig. 8, where the adaptive variables α_{ij} and β_{ij} for all nine possible communication channels converge to some finite values, and the estimated ideal weight matrices converge to the pseudo ones.

The global formation error $e_f(t) = \sqrt{\frac{1}{6}\sum_{i=1}^{6} \|e_i(t)\|^2}$ converges to zero, as shown in Fig. 9. The formation error without the NN compensation term is also shown in Fig. 9, where $e_f(t)$ does not converge to zero, and blow up to infinity at about $t = 10.5\ s$. Once more, the simplified protocol fails the TVF task.

V. Conclusions

In this paper, distributed TVF control problem is considered for a class of uncertain MASs with matching unknown nonlinearities. According to different perspectives of the dynamic coupling strengths in the network, node- and edge-based controllers are proposed, where in both controllers, NNs are embedded to identify and compensate the uncertainties online. With the help of Barbalat's lemma, both controllers are proved to drive the agents convergence to the TVF asymptotically. The protocols can also handle unknown matching bounded disturbances in the agent dynamics. Moreover, the node-based design is fully-distributed, and the edge-based one is applicable for TVF on switching graphs. For future studies, it's interesting to optimize the edge-based design to be a fully-distributed one and extend the results for MASs with directed communication topologies.

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