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Neural-network-embedded distributed average tracking of agents with matching unknown nonlinearities

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Abstract

This paper studies a distributed average tracking problem for a class of networked agents subject to heterogeneous unknown nonlinearities. The object is to design distributed protocols so as to drive these dynamic agents cooperatively tracking the average of multiple unknown signals. First, an initialize-free robust algorithm is designed for each agent incorporating a local filter, a neural network (NN) compensator, and state-dependent coupling gains with its neighbors. Here the filter is crucial for seeking the average of multiple references signals and is necessary due to the existence of uncertainties in the agents' dynamics. Then, by using adaption schemes, the algorithm is extended to a dynamic version releasing the requirement of certain global information such as the eigenvalues of the network Laplacian and the NN approximation errors. Both algorithms are rigorously proved to guarantee asymptotical average tracking with the help of well-designed Lyapunov candidates. Finally, two illustrative examples are provided to validate the theoretical results.

KEYWORDS

adaptive control, average tracking, distributed protocol, neural network

1 | INTRODUCTION

Distributed cooperative control of networked multiple agents (also known as multi-agent systems, MASs) has aroused much research enthusiasm for scientists as well as engineers recently, due to its vast applications including, but not limited to, mobile robots, wireless networks, and smart grid, see [1-3] for surveys. According to the target of distributed control in specific applications, self-organized consensus control [4,5], leader-following tracking control [6-8], and multi-leader containment problems [9,10] constitute significantly to existing research.

Beyond the above scopes, the distributed average tracking (DAT) problem [11-17] has been a topic of interest recently. It intends to design a distributed protocol for multiple agents to have their physical states cooperatively track the average of local available time-varying reference signals. The problem arises from some practical requirements such as formation control [18] and distributed optimization [19]. Among these works, [12] considers the DAT algorithms for Euler-Lagrange systems, where bounded errors can be achieved for reference signals with bounded derivatives. [13] concerns double-integrator MASs, where it is proved that DAT can be realized with reduced requirement on velocity measurements. For external signals generated by linear systems, [14] proposes a class of DAT algorithms in an edge-based framework, where continuous approximations for signum functions are used to reduce chattering. [17] studies the DAT problem for second-order agents with heterogeneous unknown nonlinear dynamics, where local filter and state-dependent gains are used to compensate the nonlinear terms by assuming that some Lipschitz-like conditions are satisfied.

When considering the relationships of the DAT problem with other technical issues in distributed systems, it can be easily understood as one of the extensions of the consensus as well as the tracking problem [4-8]. Besides, it is indeed a special class of containment problems [9,10,20] with the same number of the leaders (external signals) as that of the followers (physical agents), one-and-one communication between leaders and followers, and the final consensus state among the followers being the average of the states of the leaders. Moreover, noting that the design of DAT algorithms depends on specific agent dynamics, the DAT problem is also a variation of the dynamic average consensus (DAC) problem [21-23], which can be treated as the DAT for integrator agents and is undoubtedly another important basis research field of the DAT problem.

In the field of nonlinear control, neural network (NN) based methods have shown great efficiency, portability, and robustness in various applications, especially when the system suffers from unknown nonlinearities [24-32]. In [24-27], the tracking problems in the presence of unknown nonlinearities are studied for integrator, second-order integrator, higher-order integrator, and higher-order affine nonlinear MASs, respectively, incorporating NN controllers. The authors of [28] consider the tracking problem for a more general linear system with matching unknown nonlinearities, and also focus on the applications of NN in the formation maneuvering of marine vehicles in [29]. The consensus control for a class of nonlinear time delay MASs is considered in [30], where the approximation character of radial basis function neural networks (RBFNNs) is used to neutralize the unknown nonlinearities in the agents. In the next, [31] extends the idea to the observer based tracking problem with a high-dimensional leader, where the followers suffer from unknown nonlinear dynamics as well as time-varying delays. It should be stated that all the consensus (tracking) errors in the above literature are proved to be uniformly ultimately bounded (UUB). Recently, by introducing novel pseudo ideal weight matrices for NN controllers, [32] studies the neuro-adaptive containment problem for uncertain MASs, where asymptotic containment errors are theoretically guaranteed by Barbalat's lemma. Nevertheless, there are few concerns about NN based DAT algorithms for agents with unknown dynamics, thus to fill this gap is one main motivation of this work.

In this paper, the DAT problem for a class of linear MASs with matching unknown nonlinearities is studied, where the time-varying external signals have bounded inputs. The main challenge lies in that one must guarantee the consensus in the presence of completely unknown dynamics and the final consensus state being the average of the references signals at the same time, which can not be reached by simply designing the consensus controller as that in [4] or [24]. Besides, due to the particularities of the DAT problem as discussed before, general containment protocols in [9,10] can not be directly applied either. Despite all these difficulties, the paper firstly designs a DAT protocol consisting of a local filter, a neural network estimator, and a class of symmetric coupling weights. The protocol is then combined with adaptive methods to release the dependence of certain parameters on some global algebraic information including the smallest eigenvalue of the communication Laplacian. Both protocols guarantee the asymptotic stability of the DAT errors by leveraging the notion of pseudo converge matrices for NN estimators [32], while only UUB residual errors are realized in classical NN-based methods in [24-30]. Compared with recent works [14,16-18], the main contribution of this paper is that the dynamics of the agents are more general heterogenous nonlinear systems without Lipschitz-like conditions. In this context, the NN-based DAT algorithms are well developed and numerically testified.

The remainder of this paper is organized as follows. In Section 2, some preliminaries and problem formulation are presented. In Sections 3 and 4, two protocols for DAT are designed and analyzed in detail, which constitute the main results of the paper. In Section 5, two illustrative examples are provided. Section 6 finally concludes and outlines some potential topics for future research.

2 | PRELIMINARIES AND PROBLEM FORMULATION

2.1 | Notations and graph theory

Let $\mathbb{R}$, $\mathbb{R}^n$, $\mathbb{R}^{n\times p}$ denote the sets of real scalars, n-dimensional real vectors, $n \times p$ real matrices, respectively. Let $\mathbf{1}_n$ (resp. $\mathbf{0}_n$) denote the vector of n ones (resp. zeros) and $\boldsymbol{I}_n$ (resp. $\boldsymbol{O}_n$) denote the $n\times n$ identity (resp. zero) matrix, where the subscripts will be omitted when clear from the context. For a vector $x = (x_1, \ldots, x_n)^T \in \mathbb{R}^n$, $\|x\| = \sqrt{x^T x}$, $\|x\|_1 = \sum_{i=1}^{n} |x_i|$ and $\|x\|_\infty = \max_{i\in\{1,\ldots,n\}} |x_i|$ denote, respectively, the standard Euclidean norm, 1-norm and ∞-norm. For a square matrix A, $\mathrm{tr}(A)$ denotes its trace and $A > 0$ (resp. $A \geq 0$) means that A is positive definite (resp. semi-definite). We denote by

$\|B\|_F = \sqrt{\mathrm{tr}(B^T B)}$ the Frobenius norm of a matrix B which is not necessarily square. For local variables $x_1, x_2, \dots, x_n$, $\mathrm{col}(x_1, \dots, x_n) = (x_1^T, \dots, x_n^T)^T$ denotes the aggregated one. In this paper, diag(·) is the diagonal operator, sgn(·) is the component-wise signum function and ⊗ represents the Kronecker product.

Next, some basic concepts in graph theory are reviewed. An undirected graph $\mathcal{G}$ is specified by a node set $\mathcal{V}$ and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, where an undirected edge between node pair $i, j \in \mathcal{V}$ is denoted by (i,j). An undirected path on from vertex v_1 to v_s corresponds to a sequence of ordered edges $(v_p, v_{p+1}), p = 1, 2, \dots, s-1$. $\mathcal{G}$ is said to be connected if there exists a path between each pair of distinct nodes. The degree matrix D contains the degree of each node on its diagonal. The adjacency matrix $\mathcal{A} = (a_{ij}) \in \mathbb{R}^{|\mathcal{V}|\times|\mathcal{V}|}$ of $\mathcal{G}$ is defined by a symmetric $\{0, 1\}$-matrix, such that $a_{ij} = 1$ if and only if $(i, j) \in \mathcal{E}$. The incidence matrix $E = (E_{ik}) \in \mathbb{R}^{|\mathcal{V}|\times|\mathcal{E}|}$ of $\mathcal{G}$ (with a specific orientation) is defined by a $\{0, \pm 1\}$-matrix, such that $E_{ik} = 1$ if the vertex i is the head of the edge k, -1 if i is the tail of k, and 0 otherwise. The Laplacian matrix $\mathcal{L}$ of $\mathcal{G}$ is defined by $\mathcal{L} = D - \mathcal{A}$. Then one has $\mathcal{L} = EE^T$. In this paper, $\mathcal{G}$ is regarded as a simple graph where multiple edges and self-loops are not permitted.

For a connected graph with N nodes, $\mathcal{L}$ has a simple eigenvalue $\lambda_1 = 0$ with $\mathbf{1}_N$ as a corresponding right eigenvector, and the rest of the eigenvalues have positive real parts. Define $R \in \mathbb{R}^{N\times(N-1)}$ such that

$$\begin{bmatrix} \frac{1}{\sqrt{N}}\mathbf{1}_N^T \\ R^T \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{N}}\mathbf{1}_N, R \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{N}}\mathbf{1}_N, R \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{N}}\mathbf{1}_N^T \\ R^T \end{bmatrix} = \boldsymbol{I}_N. \tag{1}$$

Then $R^T R = \boldsymbol{I}_{N-1}$ and $RR^T = \Xi$, here $\Xi = \boldsymbol{I}_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T$, and

$$0 < \lambda_2 \boldsymbol{I}_{N-1} \leq R^T \mathcal{L} R \leq \lambda_N \boldsymbol{I}_{N-1} \tag{2}$$

where λ_2 and λ_N are the smallest non-zero eigenvalue and maximum eigenvalue of $\mathcal{L}$, respectively. Moreover, one has the equalities $\mathcal{L}\Xi = \Xi\mathcal{L} = \mathcal{L}$, $\Xi^2 = \Xi$ and $\Xi E = E$.

2.2 | Problem statement

Consider a class of network systems comprising N agents described by

$$\dot{x}_i = Ax_i + B(f_i(x_i) + u_i) \tag{3}$$

where $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^m$ are the state and control input of agent i, respectively. A, B are known matrices with compatible dimensions such that (A, B) is stabilizable. $f_i(x_i) : \mathbb{R}^n \mapsto \mathbb{R}^m$ is the unknown nonlinear dynamics of agent i. Note the agents are actually heterogeneous as $f_i(\cdot)$ maybe different from one to another. Standard assumptions for the existence of unique solutions are made.

Suppose that each agent i in the above network has access to a time-varying input signal which is generated by

$$\dot{r}_i = Ar_i + Bg_i, \tag{4}$$

where g_i is a function of t, which can be viewed as the time-varying external input of signal r_i.

The objectives concerned in this article are to design distributed controllers for agents in (3), so as to drive them to seek the average of their accessing external signals in (4), that is, $\lim_{t\to\infty} \|x_i - \frac{1}{N}\sum_{j=1}^{N} r_j\| = 0, \forall i = 1, \dots, N$.

Remark 1. The above description formulates the DAT problem. Note that the MAS model in (3) is quite general, which covers a first-order nonlinear one in [24], second-order nonlinear ones in [25] and [17], a higher-order nonlinear one in [26], when (A, B) are chosen properly, and linear time invariant ones when $f_i(x_i) \equiv 0$. As in [33,34], the system matrices of the agents and external signals remain the same in this paper, and more general cases with different ones will be our future concerns.

Assumption 1. The communication topology among N agents $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ is undirected and connected.

Assumption 2. The external signal r_i and the function g_i are known to be bounded, that is, $\|r_i\|_\infty \leq r_M$ and $\|g_i\|_\infty \leq g_M$, with $r_M, g_M > 0$.

In the following, some useful lemmas are presented for the convenience of analysis.

Lemma 1 ([17]). *Under Assumption 1, for any vector $x \in \mathbb{R}^N$, one has $x^T \mathcal{L} E \Gamma sgn(E^T x) \geq \lambda_2 x^T E \Gamma sgn(E^T x)$, where Γ is a diagonal positive definite matrix.*

Lemma 2 ([35], Barbalat lemma). *Let $h : \mathbb{R}^+ \to \mathbb{R}$ be a uniformly continuous function. Suppose that $\lim_{t\to\infty} \int_0^t h(\tau)d\tau$ exists and is finite, then $h(t) \to 0$ as $t \to \infty$.*

Before moving on, the following discussion is necessary.

2.3 | Approximation of $f_i(\cdot)$ using NN

A neural network is used to approximate the unknown $f_i(x_i)$ of agent i. Let $f_i(x_i)$ be smooth on a compact set $\Omega_x \subset \mathbb{R}^n$, assume that there exist an ideal weight matrix so that

$$f_i(x_i) = W_i^T S_i(x_i) + \epsilon_i \quad \forall x_i \in \Omega_x \tag{5}$$

with $W_i \in \mathbb{R}^{h\times m}$ the ideal weight matrix and $S_i(x_i) : \mathbb{R}^m \mapsto \mathbb{R}^h$ a vector collection of the activation functions in the hidden layer including h neurons. ϵ_i is the approximation error satisfying $\|\epsilon_i\| \leq \epsilon_{iM}$ with arbitrarily pre-designed error bound $\epsilon_{iM} > 0$ as long as h is chosen large enough, which can be guaranteed by Stone-Weierstrass approximation theorem [36].

The estimate of $f_i(x_i)$ is denoted by

$$\hat{f}_i(x_i) = \hat{W}_i^T(t)S_i(x_i), \tag{6}$$

where $\hat{W}_i(t)$ is the time-varying weight matrix to get close to W_i and will be designed later. Note that in practice, one can reach neither the ideal NN weight nor the zero functional approximation error [27].

Define $x = \text{col}(x_1, \dots, x_N)$, $f(x) = \text{col}(f_1(x_1), \dots, f_N(x_N))$, $W = \text{diag}(W_1, \dots, W_N)$, $S(x) = \text{col}(S_1(x_1), \dots, S_N(x_N))$, $\epsilon = \text{col}(\epsilon_1, \dots, \epsilon_N)$, $\hat{f}(x) = \text{col}(\hat{f}_1(x_1), \dots, \hat{f}_N(x_N))$, $\hat{W} = \text{diag}(\hat{W}_1, \dots, \hat{W}_N)$, Then, (5) and (6) can be written as

$$f(x) = W^T S(x) + \epsilon, \tag{7}$$

$$\hat{f}(x) = \hat{W}^T(t)S(x). \tag{8}$$

Furthermore, one has

$$\|\epsilon\|_\infty \le \epsilon_M, \tag{9}$$

for some threshold $\epsilon_M > 0$. Define $\tilde{W} = \hat{W} - W$ the diagonal matrix with elements $\tilde{W}_i = \hat{W}_i - W_i$ being the NN weight approximation errors.

Some standard assumptions are presented below for the convenience of analysis.

Assumption 3. [24-27,37]

- Unknown ideal NN weight matrix W_i is bounded $\forall i$, so that $\|W\|_F \le W_M$.
- NN activation functions S_i are bounded $\forall i$, so that $\|S(x)\|_F \le S_M$.

3 | DISTRIBUTED AVERAGE TRACKING PROTOCOL

In this section, an overall design process is presented and proved to solve the DAT problem.

Consider the following distributed protocol for agent i:

$$\begin{aligned} u_i =& K(x_i - r_i) + c\,\text{sgn}(K(x_i - p_i)) \\ &+ \sum_{j\in\mathcal{N}_i} c_{ij}\text{sgn}(K(p_i - p_j)) - \hat{W}_i^T S_i(x_i) \end{aligned} \tag{10}$$

where

$$\begin{aligned} p_i =& z_i + r_i \\ \dot{z}_i =& Az_i + B(K(p_i - r_i) + \sum_{j\in\mathcal{N}i} c_{ij}\text{sgn}(K(p_i - p_j))), \end{aligned} \tag{11}$$

$$c_{ij} = \alpha(\|r_i\|_\infty + \|r_j\|_\infty) + \beta \tag{12}$$

and

$$\begin{aligned} \dot{\hat{W}}_i =& \tau_i[S_i(x_i)(x_i - p_i)^T P^{-1}B - \sigma(\hat{W}_i - \bar{W}_i)] \\ \dot{\bar{W}}_i =& \pi_i\sigma(\hat{W}_i - \bar{W}_i). \end{aligned} \tag{13}$$

Here p_i is a distributed filter and z_i is an internal state, c_{ij} are time-varying coupling strengths, $\bar{W}_i$ is a pseudo ideal NN weight matrix. K is a feedback gain matrix, $c, \alpha, \beta, \tau_i, \sigma, \pi_i$ are positive scalars and $P > 0$ is a positive definite matrix. These parameters will be designed later.

Remark 2. Here the positive scalar c represents the coupling strength between physical agents and their local filters, c_{ij} with parameters α and β describing the local coupling strength between filters p_i and p_j, and is proportional to the scales of r_i and r_j. Note that c_{ij} admits certain symmetric property, that is, $c_{ij} = c_{ji}$, which is commonly used in related literature [38,39]. Moreover, it should be mentioned that the design of the protocol is partly inspired by [34] and [32].

Algorithm 1.
1. Let $\theta > 0$, solve the following linear matrix inequality (LMI):

$$AP + PA^T - 2BB^T + \theta P \le 0 \tag{14}$$

to obtain a matrix $P > 0$;
2. Set $K = -B^T P^{-1}$;
3. Choose $c > g_M + \epsilon_M, \alpha > \frac{\|B^T P^{-1}\|_\infty}{\lambda_2}, \beta > \frac{g_M}{\lambda_2}$, $\tau_i, \pi_i, \sigma > 0$, respectively.

Now we are at the position to give our first theorem.

Theorem 1. *Under Assumptions 1,2, and 3, the DAT problem can be solved if, for each agent i, designing protocol (10) and parameters following Algorithm 1. Moreover, each estimated NN weight matrix $\hat{W}_i$ converges to the corresponding pesudo ideal weight matrix $\bar{W}_i$.*

Proof. Let $\tilde{x}_i = x_i - p_i$ and $\xi = \begin{bmatrix} \xi_1 \\ \xi_{2:N} \end{bmatrix} = \left(\begin{bmatrix} \frac{1}{\sqrt{N}}\mathbf{1}_N^T \\ R^T \end{bmatrix} \otimes \boldsymbol{I}_n \right)(p - \mathbf{1}_N \otimes p^*)$, where $p^* = \frac{1}{N}\sum_{j=1}^N p_j$.

Denote $\tilde{x} = \text{col}(\tilde{x}_1, \dots, \tilde{x}_N)$, $p = \text{col}(p_1, \dots, p_N)$, $r = \text{col}(r_1, \dots, r_N)$, $g = \text{col}(g_1, \dots, g_N)$. $\mathcal{M}_e = \text{diag}(c_{ij}(t)) \in \mathbb{R}^{|\mathcal{E}|\times|\mathcal{E}|}$, where the coupling weight for each $(i, j) \in \mathcal{E}$ is recorded only once following the order of column indices in E, which is available since $c_{ij}(t) = c_{ji}(t)$ from (12).

From (3), (4), (5), and the protocols (10), one can obtain that

$$\begin{aligned} \dot{\tilde{x}} =& (\boldsymbol{I}_N \otimes (A + BK))\tilde{x} + c(\boldsymbol{I}_N \otimes B)\text{sgn}((\boldsymbol{I}_N \otimes K)\tilde{x}) \\ &- (\boldsymbol{I}_N \otimes B)(\tilde{W}^T S(x) + g - \epsilon) \end{aligned} \quad e \tag{15}$$

and

$$\begin{aligned} \dot{\xi}_1 =& \mathbf{0} \\ \dot{\xi}_{2:N} =& (\boldsymbol{I}_{N-1} \otimes (A + BK))\xi_{2:N} \\ &- (R^T \otimes BK)r + (R^T \otimes B)g \\ &+ (R^T E\mathcal{M}_e \otimes B)\text{sgn}((E^T R \otimes K)\xi_{2:N}). \end{aligned} \tag{16}$$

Besides, it is clear that $\dot{\xi}_1 = \mathbf{0}$, which indicates $\xi_1 \equiv \mathbf{0}$.

Consider the following Lyapunov function candidate:

$$V_1 = V_x + V_\xi + V_w \tag{17}$$

where

$$\begin{aligned} V_x &= \tilde{x}^T(\boldsymbol{I}_N \otimes P^{-1})\tilde{x} \\ V_\xi &= \xi_{2:N}^T(R^T\mathcal{L}R \otimes P^{-1})\xi_{2:N} \\ V_w &= \mathrm{tr}(\tilde{W}^T\tau^{-1}\tilde{W}) + \mathrm{tr}(\tilde{\tilde{W}}^T\pi^{-1}\tilde{\tilde{W}}). \end{aligned} \tag{18}$$

Here $P > 0$ is a solution of LMI (14), R is defined in (1), $\tau = \mathrm{diag}(\tau_1, \dots, \tau_N)$, $\tilde{\tilde{W}} = \mathrm{diag}(\tilde{\tilde{W}}_1, \dots, \tilde{\tilde{W}}_N)$ with $\tilde{\tilde{W}}_i = \bar{W}_i - W_i$ and $\pi = \mathrm{diag}(\pi_1, \dots, \pi_N)$.

Firstly, from (15), one has

$$\begin{aligned} \dot{V}_x &= 2\tilde{x}^T(\boldsymbol{I}_N \otimes P^{-1})\dot{\tilde{x}} \\ &= \tilde{x}^T(\boldsymbol{I}_N \otimes (P^{-1}(A+BK) + (A+BK)^TP^{-1}))\tilde{x} \\ &\quad + 2c\tilde{x}^T(\boldsymbol{I}_N \otimes P^{-1}B)\mathrm{sgn}((\boldsymbol{I}_N \otimes K)\tilde{x}) \\ &\quad - 2\tilde{x}^T(\boldsymbol{I}_N \otimes P^{-1}B)(\tilde{W}^TS(x) + g - \epsilon). \end{aligned} \tag{19}$$

Substitute K in Algorithm (1) into the above, one has

$$\begin{aligned} \dot{V}_x &= \tilde{x}^T(\boldsymbol{I} \otimes P^{-1})(\boldsymbol{I} \otimes (AP + PA^T - 2BB^T))(\boldsymbol{I} \otimes P^{-1})\tilde{x} \\ &\quad - 2c\tilde{x}^T(\boldsymbol{I} \otimes P^{-1}B)\mathrm{sgn}((\boldsymbol{I} \otimes B^TP^{-1})\tilde{x}) \\ &\quad - 2\tilde{x}^T(\boldsymbol{I} \otimes P^{-1}B)(\tilde{W}^TS(x) + g - \epsilon) \\ &\le -\theta\tilde{x}^T(\boldsymbol{I} \otimes P^{-1})\tilde{x} - 2\sum_{i=1}^N c\|B^TP^{-1}\tilde{x}_i\|_1 \\ &\quad - 2\tilde{x}^T(\boldsymbol{I} \otimes P^{-1}B)(\tilde{W}^TS(x) + g - \epsilon) \\ &\le -\theta\tilde{x}^T(\boldsymbol{I} \otimes P^{-1})\tilde{x} - 2c\sum_{i=1}^N \|B^TP^{-1}\tilde{x}_i\|_1 \\ &\quad - 2\tilde{x}^T(\boldsymbol{I} \otimes P^{-1}B)(\tilde{W}^TS(x)) \\ &\quad + 2(\|g\|_\infty + \|\epsilon\|_\infty)\|(\boldsymbol{I} \otimes B^TP^{-1})\tilde{x}\|_1 \end{aligned} \tag{20}$$

where LMI (14) is used here to get the second inequality and Hölder inequality is used to get the last one. Note the fact that $x^T\mathrm{sgn}(x) = \|x\|_1$ for an arbitrary real column vector x. It then follows Assumption 2 and (9) that

$$\begin{aligned} \dot{V}_x &\le -\theta\tilde{x}^T(\boldsymbol{I}_N \otimes P^{-1})\tilde{x} - 2\tilde{x}^T(\boldsymbol{I}_N \otimes P^{-1}B)(\tilde{W}^TS(x)) \\ &\quad - 2\sum_{i=1}^N(c - (\|g\|_M + \|\epsilon\|_M))\|B^TP^{-1}\tilde{x}_i\|_1. \end{aligned} \tag{21}$$

Then, from (16), one has

$$\begin{aligned} \dot{V}_\xi &= 2\xi_{2:N}^T(R^T\mathcal{L}R \otimes P^{-1})\dot{\xi}_{2:N} \\ &= \xi_{2:N}^T(R^T\mathcal{L}R \otimes (P^{-1}(A+BK) \\ &\qquad + (A+BK)^TP^{-1}))\xi_{2:N} \\ &\quad -2\xi_{2:N}^T(R^T\mathcal{L} \otimes P^{-1}BK)r + 2\xi_{2:N}^T(R^T\mathcal{L} \otimes P^{-1}B)g \\ &\quad +2\xi_{2:N}^T(R^T\mathcal{L}E\mathcal{M}_e \otimes P^{-1}B)\mathrm{sgn}((E^TR \otimes K)\xi_{2:N}). \end{aligned} \tag{22}$$

Similarly, one has

$$\begin{aligned} \dot{V}_\xi &\le -\theta\xi_{2:N}^T(R^T\mathcal{L}R \otimes P^{-1})\xi_{2:N} \\ &\quad + 2\xi_{2:N}^T(R^T\mathcal{L} \otimes P^{-1}BB^TP^{-1})r \\ &\quad + 2\xi_{2:N}^T(R^T\mathcal{L} \otimes P^{-1}B)g \\ &\quad -2\xi_{2:N}^T(R^T\mathcal{L}E\mathcal{M}_e \otimes P^{-1}B)\mathrm{sgn}((E^TR \otimes B^TP^{-1})\xi_{2:N}) \\ &\le -\theta\xi_{2:N}^T(R^T\mathcal{L}R \otimes P^{-1})\xi_{2:N} \\ &\quad +2\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\|B^TP^{-1}r_i\|_\infty\|B^TP^{-1}(\varphi_i - \varphi_j)\|_1 \\ &\quad +2\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\|g_i\|_\infty\|B^TP^{-1}(\varphi_i - \varphi_j)\|_1 \\ &\quad -2\lambda_2\varphi^T(E\mathcal{M}_e \otimes P^{-1}B)\mathrm{sgn}((E^T \otimes B^TP^{-1})\varphi) \end{aligned} \tag{23}$$

where LMI (14) is again used to get the first inequality, and Hölder inequality and Lemma 1 are used to get the last one. Here $\varphi = \mathrm{col}(\varphi_1, \dots, \varphi_N) = (R \otimes \boldsymbol{I}_n)\xi_{2:N}$. Note that

$$\begin{aligned} &2\varphi^T(E\mathcal{M}_e \otimes P^{-1}B)\mathrm{sgn}((E^T \otimes B^TP^{-1})\varphi) \\ &\quad = \sum_{i=1}^N\sum_{j\in\mathcal{N}_i}c_{ij}\|B^TP^{-1}(\varphi_i - \varphi_j)\|_1, \end{aligned} \tag{24}$$

then it follows (12), (23), (24), and Assumption 2 that

$$\begin{aligned} \dot{V}_\xi &\le -\theta\xi_{2:N}^T(R^T\mathcal{L}R \otimes P^{-1})\xi_{2:N} \\ &\quad + 2\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\|B^TP^{-1}r_i\|_\infty\|B^TP^{-1}(\varphi_i - \varphi_j)\|_1 \\ &\quad + 2\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}g_\infty\|B^TP^{-1}(\varphi_i - \varphi_j)\|_1 \\ &\quad - 2\lambda_2\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}c_{ij}\|B^TP^{-1}(\varphi_i - \varphi_j)\|_1 \\ &\le -\theta\xi_{2:N}^T(R^T\mathcal{L}R \otimes P^{-1})\xi_{2:N} \\ &\quad - 2\sum_{i=1}^N\sum_{j\in\mathcal{N}_i}\big((\alpha\lambda_2 - \|B^TP^{-1}\|_\infty)\|r_i\|_\infty \\ &\quad +(\beta\lambda_2 - g_M)\big)\|B^TP^{-1}(\varphi_i - \varphi_j)\|_1. \end{aligned} \tag{25}$$

Moreover, given (13), one can obtain that

$$\begin{aligned} \dot{V}_w &= 2\sum_{i=1}^N\mathrm{tr}(\frac{1}{\tau_i}\tilde{W}_i^T\dot{\tilde{W}}_i) + 2\sum_{i=1}^N\mathrm{tr}(\frac{1}{\pi_i}\tilde{\tilde{W}}_i^T\dot{\tilde{\tilde{W}}}_i) \\ &= 2\sum_{i=1}^N\mathrm{tr}(\tilde{W}_i^TS_i(x_i)\tilde{x}_i^TP^{-1}B) \\ &\quad - 2\sigma\sum_{i=1}^N\mathrm{tr}((\hat{W}_i - \bar{W}_i)^T(\hat{W}_i - \bar{W}_i)) \\ &\le 2\sum_{i=1}^N\mathrm{tr}(\tilde{W}_i^TS_i(x_i)\tilde{x}_i^TP^{-1}B). \end{aligned} \tag{26}$$

Finally, it follows (21), (25), (26), and the parameters chosen from 3) in Algorithm (1), one has

$$\begin{aligned}\dot{V}_1 \leq& -\theta\tilde{x}^T(\boldsymbol{I}_N\otimes P^{-1})\tilde{x} - 2\tilde{x}^T(\boldsymbol{I}_N\otimes P^{-1}B)(\tilde{W}^TS(x))\\ &-\theta\xi_{2:N}^T(R^T\mathcal{L}R\otimes P^{-1})\xi_{2:N}\\ &+2\sum_{i=1}^{N}\mathrm{tr}(\tilde{W}_i^TS_i(x_i)\tilde{x}_i^TP^{-1}B)\\ \leq& -\theta(\tilde{x}^T(\boldsymbol{I}_N\otimes P^{-1})\tilde{x} + \xi_{2:N}^T(R^T\mathcal{L}R\otimes P^{-1})\xi_{2:N})\\ =& -\theta[\tilde{x}^T,\xi_{2:N}^T]\begin{bmatrix}\boldsymbol{I}_N\otimes P^{-1} & \boldsymbol{O}\\ \boldsymbol{O} & R^T\mathcal{L}R\otimes P^{-1}\end{bmatrix}\begin{bmatrix}\tilde{x}\\ \xi_{2:N}\end{bmatrix}\\ \triangleq& -\theta e^T\Phi e \leq 0\end{aligned} \tag{27}$$

where the fact that $\mathrm{tr}(XY) = \mathrm{tr}(YX)$ for any compatible matrices X, Y is used to get the second inequality.

It is clear that $\boldsymbol{\Phi} > 0$, so $V_1(t)$ is nonincreasing, which guarantees that all the signals $\tilde{x}$, $\xi_{2:N}$, $\tilde{W}_i$ and $\tilde{\bar{W}}_i$ in $V_1(t)$ are bounded. Since $V_1(t) \leq V_1(0)$ and is nonincreasing, it thus has a finite limit V_1^{∞} as $t\to\infty$. Integrate both sides of (27), one has

$$\int_0^{+\infty}\theta e^T(t)\Phi e(t)dt \leq V_1(0) - V_1^{\infty}. \tag{28}$$

Under (15), (16), and Assumptions 2 and 3, $\dot{e}$ is also uniformly bounded, then $\theta e^T\boldsymbol{\Phi}e$ is uniformly continuous. By utilizing Lemma 2, one has $\lim_{t\to\infty}\theta e^T(t)\Phi e(t) = 0$, which guarantees $\lim_{t\to\infty}\|e(t)\| = 0$ as $\boldsymbol{\Phi} > 0$, that is, $\lim_{t\to\infty}\|\tilde{x}(t)\| = 0$ and $\lim_{t\to\infty}\|\xi_{2:N}(t)\| = 0$, then $\lim_{t\to\infty}\|\xi(t)\| = 0$. From the definitions of $\tilde{x}$ and ξ, one has

$$\lim_{t\to\infty}\|x_i - p_i\| = 0, \quad \lim_{t\to\infty}\|p_i - p^*\| = 0. \tag{29}$$

Then we prove the solvability of the protocol for DAT by showing

$$\lim_{t\to\infty}\|p^* - \frac{1}{N}\sum_{i=1}^{N}r_i\| = 0 \tag{30}$$

with an exponential decaying rate. In fact, denote $\zeta(t) = p^* - \frac{1}{N}\sum_{i=1}^{N}r_i$, then it follows from (4) and (11) that

$$\dot{\zeta} = (A + BK)\zeta, \tag{31}$$

with K designed in Algorithm 1, $A + BK$ is Hurwitz.

Finally, it follows (29), (30), and triangle inequality that

$$\lim_{t\to\infty}\|x_i - \frac{1}{N}\sum_{i=1}^{N}r_i\| = 0, \tag{32}$$

that is, the DAT problem is solved.

Besides, denote $\hat{W}_i^e = \hat{W}_i - \bar{W}_i$, then it follows (13) that

$$\dot{\hat{W}}_i^e = -\sigma(\tau_i + \pi_i)\hat{W}_i^e + \tau_iS_i(x_i)\tilde{x}_i^TP^{-1}B. \tag{33}$$

Since $\lim_{t\to\infty}\|\tilde{x}_i\| = 0$, $S_i(x_i)$ is uniformly bounded, σ, τ_i and π_i are given positive constants, then the asymptotic gain property holds for (33) [40], which indicates that $\hat{W}_i^e$ is input to state stable. One then has $\lim_{t\to\infty}\|\hat{W}_i^e\|_F = 0$, that is, $\lim_{t\to\infty}\hat{W}_i = \bar{W}_i$. This completes the proof. □

Remark 3. It should be mentioned that the controller (10) is also robust to the agents exposed into matching bounded disturbances as in [28,32], which can be shown by similar steps in the proofs with a few modifications. We omit the details for brevity.

Remark 4. Note that the pseudo ideal matrices server as satisfactory replacements for the ideal ones, enabling us to drive the consensus error vector to zero asymptotically. Moreover, the results in this paper are semi-global as we assume the existence of ideal weight matrices for f_i on a compact set Ω_x. As in [27,41,42], the area of Ω_x can be arbitrarily large since it has no influence on the controller, and the results will be global if $\Omega = \mathbb{R}^n$.

Remark 5. As a matter of fact, the existence of signum function reflect the discontinuous properties of the dynamics in (15) and (16). Therefore, the stability analysis should be performed in contents of differential inclusions and nonsmooth theory. Since the signum function is measurable and essentially bounded, the solutions for (15) and (16) always exist in the sense of Filippov [43]. Note that the Lyapunov candidate (17) is continuous differentiable and its set-valued Lie derivative is a singleton at the discontinuous point, the proof still holds. More details can be found in [32,33,38] and the references therein. In this paper, we do not use differential inclusions in the proofs to avoid symbol redundancy.

4 | DYNAMIC PROTOCOL THAT RELEASES THE PARAMETERS

In the last section, the designed protocol can be executed by each agent in a distributed way. Nevertheless, the coupling parameters c, α and β in Algorithm 1 depend on some global information, such as λ_2, g_M as well as ϵ_M. To further eliminate these limitations, the following dynamic-gain protocol is considered for each agent i:

$$\begin{aligned}u_i =& K(x_i - r_i) + c_i\mathrm{sgn}(K(x_i - p_i))\\ &+\sum_{j\in\mathcal{N}_i}c_{ij}\mathrm{sgn}(K(p_i - p_j)) - \hat{W}_i^TS_i(x_i),\end{aligned} \tag{34}$$

$$
\begin{aligned}
p_i =& z_i + r_i \\
\dot{z}_i =& Az_i + B(K(p_i - r_i) + \sum_{j\in\mathcal{N}_i} c_{ij}\mathrm{sgn}(K(p_i - p_j))),
\end{aligned} \tag{35}
$$

$$
\begin{aligned}
\dot{c}_i =& \kappa_i \|K(x_i - p_i)\|_1 \\
c_{ij} =& \alpha_{ij}(\|r_i\|_\infty + \|r_j\|_\infty) + \beta_{ij},
\end{aligned} \tag{36}
$$

$$
\begin{aligned}
\dot{\alpha}_{ij} =& \mu_{ij}(\|r_i\|_\infty + \|r_j\|_\infty)\|K(\varphi_i - \varphi_j)\|_1 \\
\dot{\beta}_{ij} =& \nu_{ij}\|K(\varphi_i - \varphi_j)\|_1,
\end{aligned} \tag{37}
$$

$$
\begin{aligned}
\dot{\hat{W}}_i =& \tau_i[S_i(x_i)(x_i - p_i)^T P^{-1}B - \sigma(\hat{W}_i - \bar{W}_i)] \\
\dot{\bar{W}}_i =& \pi_i \sigma(\hat{W}_i - \bar{W}_i),
\end{aligned} \tag{38}
$$

where $\varphi = \mathrm{col}(\varphi_1, \ldots, \varphi_N) = (\Xi \otimes \boldsymbol{I}_n)p$.

The following theorem concludes this distributed strategy for the DAT problem.

Theorem 2. *Under Assumptions 1,2, and 3, the DAT problem can be solved if, for each agent i, designing protocol (34) with $K = -B^T P^{-1}$ and parameters $\kappa_i, \mu_{ij}, \nu_{ij} > 0$, here P is a solution of LMI (14). Moreover, each estimated NN weight matrix $\hat{W}_i$ converges to the corresponding pesudo ideal weight matrix $\bar{W}_i$ and each c_i, α_{ij} as well as β_{ij} will converge to some finite steady value.*

Proof. Denote $\mathcal{M}_\mathrm{n} = \mathrm{diag}(c_1, \ldots, c_N)$, then the dynamics of $\tilde{x}$ with protocol (34) can be written as

$$
\begin{aligned}
\dot{\tilde{x}} =& (\boldsymbol{I}_N \otimes (A + BK))\tilde{x} + (\mathcal{M}_\mathrm{n} \otimes B)\mathrm{sgn}((\boldsymbol{I}_N \otimes K)\tilde{x}) \\
&- (\boldsymbol{I}_N \otimes B)(\tilde{W}^T S(x) + g - \epsilon)
\end{aligned} \tag{39}
$$

and the dynamics of ξ can be written in the same form as that of (16). Note the equality $c_{ij}(t) = c_{ji}(t)$ still holds under the time-varying α_{ij} and β_{ij}.

Consider the following Lyapunov function candidate:

$$
V_2 = V'_x + V'_\xi + V_w \tag{40}
$$

where

$$
\begin{aligned}
V'_x =& \tilde{x}^T(\boldsymbol{I}_N \otimes P^{-1})\tilde{x} + \sum_{i=1}^N \frac{(c_i - \bar{c})^2}{\kappa_i} \\
V'_\xi =& \xi_{2:N}^T(R^T\mathcal{L}R \otimes P^{-1})\xi_{2:N} \\
&+ \lambda_2 \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \frac{(\alpha_{ij} - \bar{\alpha})^2}{\mu_{ij}} + \frac{(\beta_{ij} - \bar{\beta})^2}{\nu_{ij}} \\
V_w =& \mathrm{tr}(\tilde{W}^T\tau^{-1}\tilde{W}) + \mathrm{tr}(\tilde{\bar{W}}^T \pi^{-1}\tilde{\bar{W}}).
\end{aligned} \tag{41}
$$

Here $P > 0$ is a solution of LMI (14), $\bar{c} > 0$ will be determined later. $\tilde{\bar{W}}_i$, τ and π are defined as in (18).

Firstly, from (20) and $\dot{c}_i$ in (36), one can compute that

$$
\begin{aligned}
\dot{V}'_x \leq& -\theta\tilde{x}^T(\boldsymbol{I} \otimes P^{-1})\tilde{x} \\
&- 2\tilde{x}^T(\mathcal{M}_\mathrm{n} \otimes P^{-1}B)\mathrm{sgn}((\boldsymbol{I} \otimes B^T P^{-1})\tilde{x}) \\
&- 2\tilde{x}^T(\boldsymbol{I} \otimes P^{-1}B)(\tilde{W}^T S(x) + g - \epsilon) \\
&+ 2\sum_{i=1}^N (c_i - \bar{c})\|K(x_i - p_i)\|_1 \\
\leq& -\theta\tilde{x}^T(\boldsymbol{I} \otimes P^{-1})\tilde{x} - 2\sum_{i=1}^N c_i\|B^T P^{-1}\tilde{x}_i\|_1 \\
&- 2\tilde{x}^T(\boldsymbol{I} \otimes P^{-1}B)(\tilde{W}^T S(x)) \\
&+ 2(\|g\|_\infty + \|\epsilon\|_\infty)\|(\boldsymbol{I} \otimes B^T P^{-1})\tilde{x}\|_1 \\
&+ 2\sum_{i=1}^N (c_i - \bar{c})\|B^T P^{-1}\tilde{x}_i\|_1.
\end{aligned} \tag{42}
$$

Considering (9) and Assumption 2, some manipulations on (42) give that

$$
\begin{aligned}
\dot{V}'_x \leq& -\theta\tilde{x}^T(\boldsymbol{I} \otimes P^{-1})\tilde{x} - 2\tilde{x}^T(\boldsymbol{I} \otimes P^{-1}B)(\tilde{W}^T S(x)) \\
&- 2\sum_{i=1}^N (\bar{c} - (\|g\|_\infty + \|\epsilon\|_\infty))\|B^T P^{-1}\tilde{x}_i\|_1 \\
\leq& -\theta\tilde{x}^T(\boldsymbol{I} \otimes P^{-1})\tilde{x} - 2\tilde{x}^T(\boldsymbol{I} \otimes P^{-1}B)(\tilde{W}^T S(x)) \\
&- 2\sum_{i=1}^N (\bar{c} - (\|g\|_M + \|\epsilon\|_M))\|B^T P^{-1}\tilde{x}_i\|_1.
\end{aligned} \tag{43}
$$

Then, from (25) and the definitions of c_{ij} in (36), $\dot{\alpha}_{ij}$, $\dot{\beta}_{ij}$ in (37), one has

$$
\begin{aligned}
\dot{V}'_\xi \leq& -\theta\xi_{2:N}^T(R^T\mathcal{L}R \otimes P^{-1})\xi_{2:N} \\
&+ 2\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \|B^T P^{-1} r_i\|_\infty \|B^T P^{-1}(\varphi_i - \varphi_j)\|_1 \\
&+ 2\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} g_M \|B^T P^{-1}(\varphi_i - \varphi_j)\|_1 \\
&- 2\lambda_2 \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} [c_{ij}\|B^T P^{-1}(\varphi_i - \varphi_j)\|_1 \\
&- (\alpha_{ij} - \bar{\alpha})(\|r_i\|_\infty + \|r_j\|_\infty)\|B^T P^{-1}(\varphi_i - \varphi_j)\|_1 \\
&- (\beta_{ij} - \bar{\beta})\|B^T P^{-1}(\varphi_i - \varphi_j)\|_1] \\
=& -\theta\xi_{2:N}^T(R^T\mathcal{L}R \otimes P^{-1})\xi_{2:N} \\
&+ 2\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \|B^T P^{-1} r_i\|_\infty \|B^T P^{-1}(\varphi_i - \varphi_j)\|_1 \\
&+ 2\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} g_M \|B^T P^{-1}(\varphi_i - \varphi_j)\|_1 \\
&- 2\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \big(\bar{\alpha}\lambda_2(\|r_i\|_\infty + \|r_j\|_\infty) \\
&+ \bar{\beta}\lambda_2\big)\|B^T P^{-1}(\varphi_i - \varphi_j)\|_1,
\end{aligned} \tag{44}
$$

then it follows (44), Assumption 2 and some manipulations that

$$
\begin{aligned}
\dot{V}'_{\xi} \le & -\theta \xi_{2:N}^{T}(R^{T}\mathcal{L}R \otimes P^{-1})\xi_{2:N} \\
& -2\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\Big((\bar{\alpha}\lambda_2 - \|B^{T}P^{-1}\|_{\infty})\|r_i\|_{\infty} \\
& +(\bar{\beta}\lambda_2 - g_M)\Big)\,\|B^{T}P^{-1}(\varphi_i-\varphi_j)\|_1.
\end{aligned} \tag{45}
$$

Moreover, note the V_w in (40) remains the same as that in (17) and the design of $\dot{\hat{W}}_i$ in (38) is the same as in (13). By choosing sufficiently large $\bar{c}$, $\bar{\alpha}$ and $\bar{\beta}$ in (40) such that $\bar{c} > g_M + \epsilon_M, \bar{\alpha} > \frac{\|B^TP^{-1}\|_\infty}{\lambda_2}, \bar{\beta} > \frac{g_M}{\lambda_2}$, it follows (43), (45) and (26) that

$$
\begin{aligned}
\dot{V}_2 \le & -\theta \tilde{x}^{T}(\boldsymbol{I}_N \otimes P^{-1})\tilde{x} - 2\tilde{x}^{T}(\boldsymbol{I}_N \otimes P^{-1}B)(\tilde{W}^{T}S(x)) \\
& -\theta \xi_{2:N}^{T}(R^{T}\mathcal{L}R \otimes P^{-1})\xi_{2:N} \\
& +2\sum_{i=1}^{N}\mathrm{tr}(\tilde{W}_i^{T}S_i(x_i)\tilde{x}_i^{T}P^{-1}B) \\
\le & -\theta(\tilde{x}^{T}(\boldsymbol{I}_N \otimes P^{-1})\tilde{x} + \xi_{2:N}^{T}(R^{T}\mathcal{L}R \otimes P^{-1})\xi_{2:N}) \\
\triangleq & -\theta e^{T}\Phi e \le 0
\end{aligned} \tag{46}
$$

where e and $\boldsymbol{\Phi}$ are defined in (27). Then one has $V_2(t)$ is also nonincreasing, which guarantees that all the signals $\tilde{x}$, c_i, $\xi_{2:N}$, α_{ij}, β_{ij}, $\tilde{W}_i$, and $\tilde{\bar{W}}_i$ in $V_2(t)$ are bounded.

Following the same procedures in the proof of Theorem 1, one has

$$
\lim_{t\to\infty}\|x_i - \frac{1}{N}\sum_{i=1}^{N} r_i\| = 0 \tag{47}
$$

and

$$
\lim_{t\to\infty}\hat{W}_i = \bar{W}_i. \tag{48}
$$

Since $\kappa_i, \mu_{ij}, \nu_{ij} > 0$, all c_i, α_{ij} and β_{ij} are designed to be monotonically increasing, then they will all converge to some finite steady values independently, which can be guaranteed by their boundness. This completes the proof. □

Remark 6. In [11], the initial internal states are required to satisfy $\sum_{i=1}^{N} z_i(0) = 0$ and $\sum_{i=1}^{N} \dot{z}_i(0) = 0$. Neither of these conditions are required in protocol (34) as well as (10). In other words, the proposed algorithms are initialize-free.

Remark 7. Although it is theoretically proved that the DAT errors converge to zero, a practical issue when implementing the dynamic protocol (34) is that the coupling gains $c_i, \alpha_{ij}, \beta_{ij}$ may increase slowly because of measurement errors, chattering effects, or disturbances [33,34]. To handle this issue, one can introduce small scalars $T_1, T_2 > 0$ and update c_i by (36) whenever $\|x_i - p_i\|_1 > T_1$ and α_{ij}, β_{ij} by (37) whenever $\|\varphi_i - \varphi_j\|_1 > T_2$. On the contrary of either case, one can hold the corresponding coupling gains. Then, as long as the DAT errors converge into a desirable bound, the adaptive parameters will converge to some finite values.

Remark 8. In practice, the chattering phenomena under discontinuous protocols (10) and (34) can be reduced if one replace function $\mathrm{sgn}(z)$ with nonlinear function

$$
\varpi(z,t) = \frac{z}{\|z\| + \varepsilon e^{-\varrho t}} \tag{49}
$$

or simply

$$
\varpi(z) = \frac{z}{\|z\| + \varepsilon} \tag{50}
$$

where $\varepsilon, \varrho > 0$. In fact, the above are continuous time-varying and time-invariant approximations, respectively, of the signum function based on the boundary layer concept [44]. More details can be found in [14], [34] and references therein.

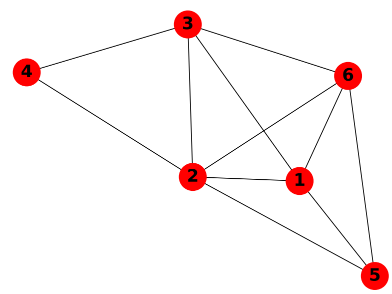

FIGURE 1 Communication topology among the agents

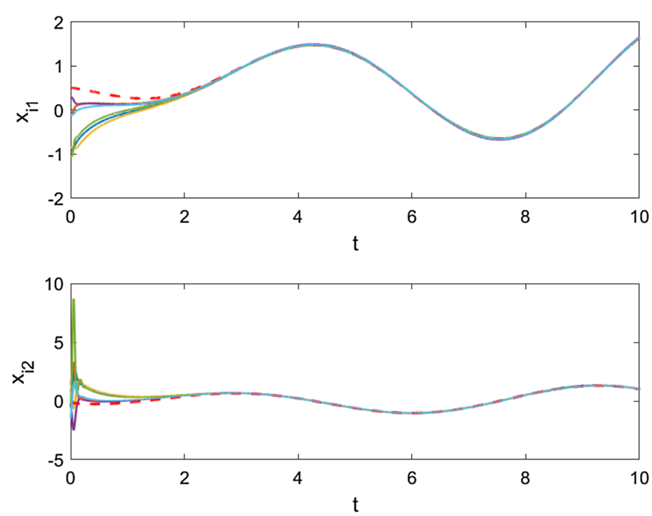

FIGURE 2 States of agents x_i (solid lines) and the average of r_i (dashed line), $i = 1, \ldots, 6$

5 | ILLUSTRATIVE EXAMPLES

In this section, two numerical examples are given to illustrate the theoretical results. In both examples, the communication topologies among the agents are assumed the same and depicted in Figure 1, and the external inputs for the signals are also the same and given by

$$g_1 = \sin(0.8t)$$
$$g_2 = 0.5\sin(0.7t) + 0.5\sin(0.6t)$$
$$g_3 = \frac{e^t}{e^t+1}$$
$$g_4 = \frac{2}{\pi}\arctan\left(\frac{\pi}{2}t\right)$$
$$g_5 = \frac{t}{1+t^2}$$
$$g_6 = \tanh(t),$$

then all g_i, $i = 1, \ldots, 6$, are bounded by $g_M = 1$. In the simulations, the initial states $r_i(0)$ are randomly chosen from

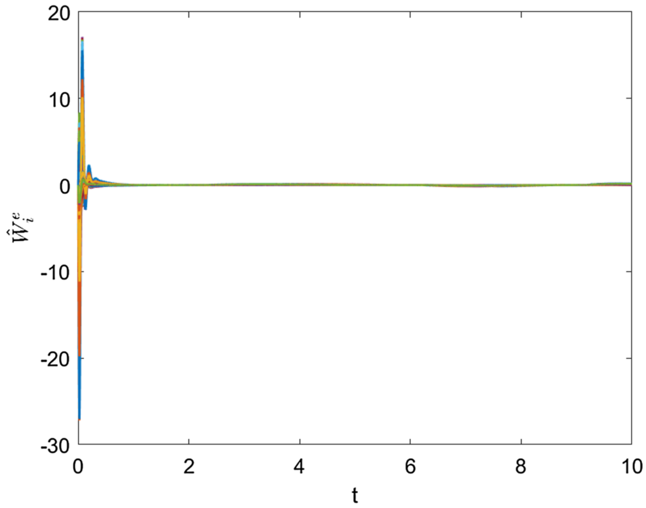

FIGURE 3 Component-wise values of $\hat{W}_i^e$, $i = 1, \ldots, 6$

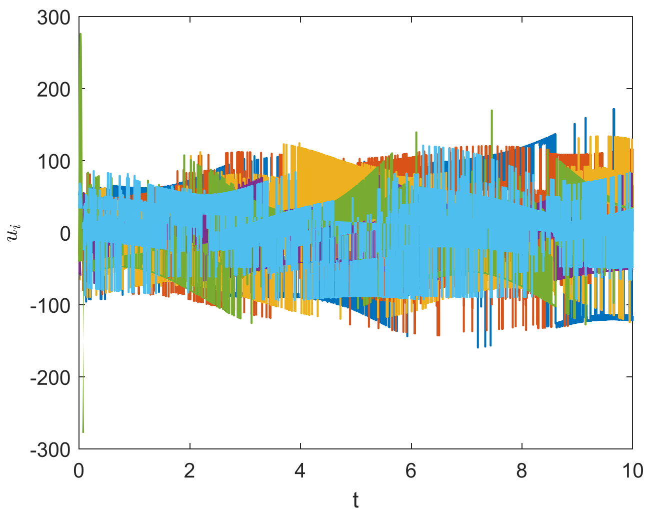

FIGURE 4 Control inputs u_i in (10), $i = 1, \ldots, 6$

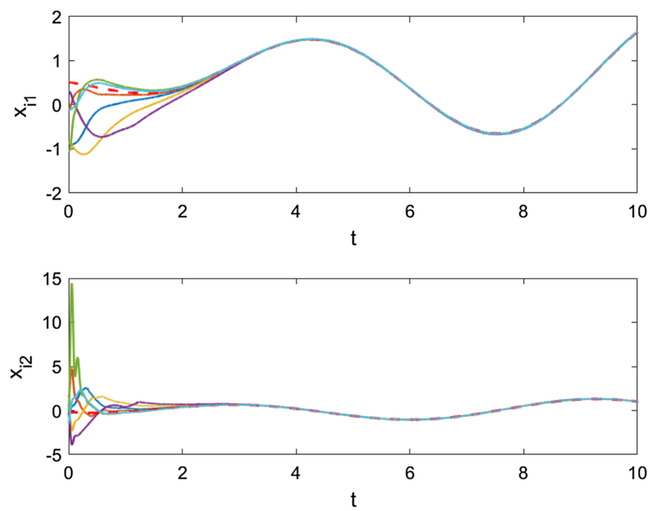

FIGURE 5 States of agents x_i (solid lines) and the average of r_i (dashed line), $i = 1, \ldots, 6$

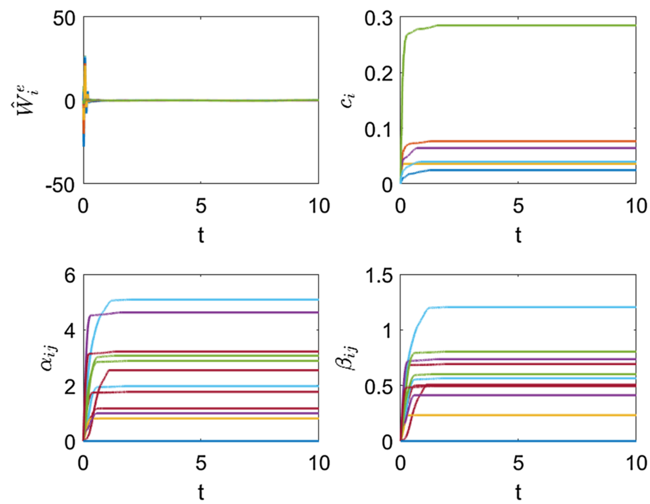

FIGURE 6 Component-wise values of $\hat{W}_i^e$ (upper left), and adaptive coupling weights c_i (upper right), α_{ij} (lower left) and β_{ij} (lower right), $i, j = 1, \ldots, 6$

normal distribution with standard derivation 2, and when implementing the protocols, $x_i(0)$, $z_i(0)$ and the first-layer weights of the NNs are initialized following standard normal distribution and $\hat{W}_i(0) = \bar{W}_i(0) = \boldsymbol{O}$. For each agent in both examples, 30 hidden neurons with Sigmoid activation functions are assigned to approximate its nonlinearities, under which we could expect sufficient approximation accuracies with bound $\epsilon_M = 1$.

Example 1. Consider the network of 6 physical agents the dynamics of which can be described by uncertain second order oscillators with

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

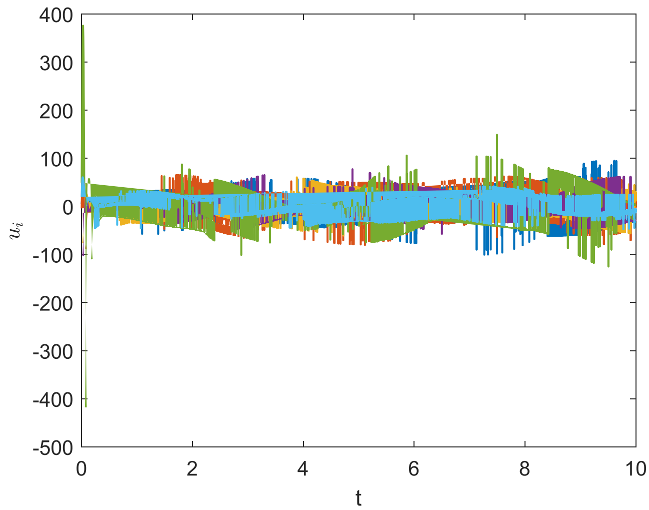

FIGURE 7 Control inputs u_i in (34), $i = 1, \ldots, 6$

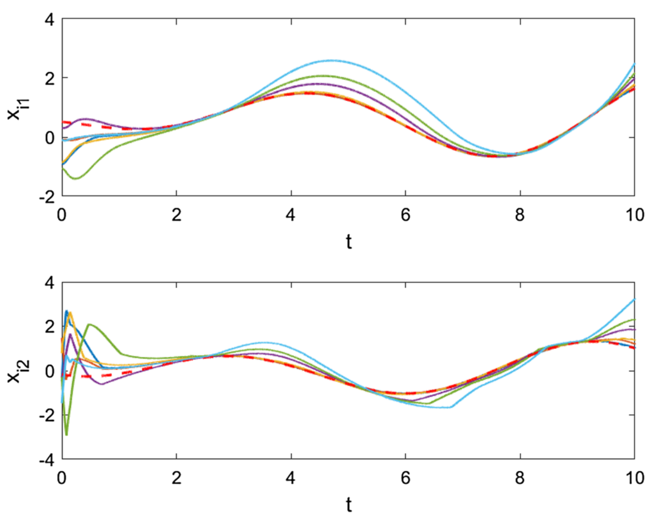

FIGURE 8 States of agents x_i (solid lines) and the average of r_i (dashed line), $i = 1, \ldots, 6$, under (10) without the last NN term

and matching unknown nonlinear functions $f_i(x_i) = ix_{i1} + x_{i2}^2$.

First, to verify Theorem 1, let $\theta = 2$ and solve LMI (14), we obtain

$$P = \begin{bmatrix} 0.1277 & -0.2066 \\ -0.2066 & 0.5409 \end{bmatrix}.$$

Set

$$K = [-7.8200, -4.8354]$$

and choose $c = 3$, $\alpha = 4.5$, $\beta = 0.65$, $\tau_i = 60$, $\pi_i = 6$, $\sigma = 0.6$ as instructed by Algorithm 1. Then all conditions of Theorem 1 are satisfied. After implementing protocol (10), the states of all individuals are visualized in Figure 2, the component-wise values of pseudo converge errors $\hat{W}_i^e$ are provided in Figure 3, and the control inputs are shown in Figure 4. It can be seen

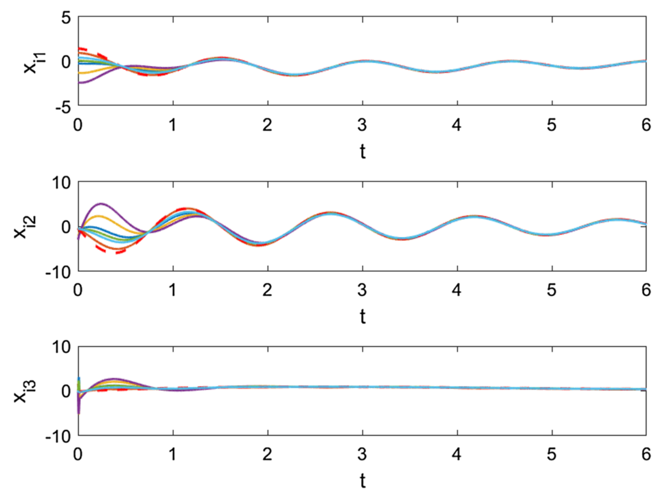

FIGURE 9 States of agents x_i (solid lines) and the average of r_i (dashed line), $i = 1, \ldots, 6$

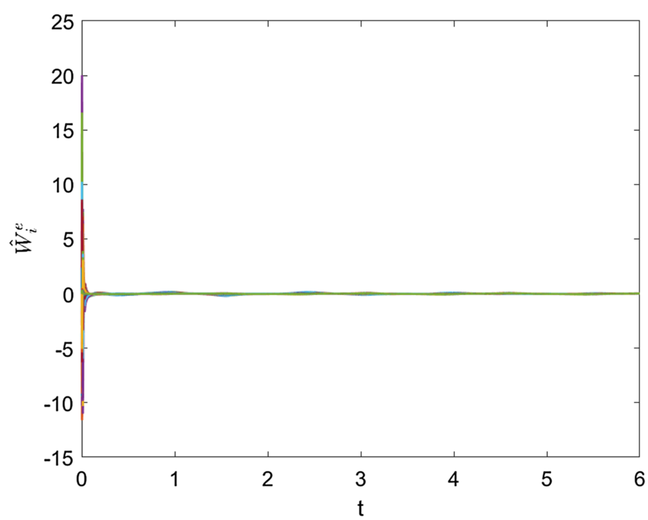

FIGURE 10 Component-wise values of $\hat{W}_i^e$, $i = 1, \ldots, 6$

that all agents indeed track the average of the reference signals.

Then, to verify Theorem 2, dynamic protocol (34) is implemented for the network system with the same initial conditions and NN parameters as above and $\kappa_i = \mu_{ij} = \nu_{ij} = 0.1$. Considering Remark 7, choose $T_1 = T_2 = 0.6$. Then the states of all agents are provided in Figure 5, which shows the distributed average tracking is realized. The component-wise values of $\hat{W}_i^e$ and dynamical coupling weights c_i, α_{ij}, β_{ij} are shown in Figure 6, where all the dynamical coupling weights converge to some steady values after about 3 seconds. Finally the control inputs are provided in Figure 7.

In order to highlight the necessity of the incorporation of NN, we turn off the NN approximation term $\hat{W}_i^T S_i(x_i)$ in scheme (10) and leaving only the first three feedback terms, which is analogous to the controller studied in [34]. Then the evolutions of the agents are shown in Figure 8. Clearly, this simplified strategy fails to solve the DAT problem.

Example 2. Consider the network of 6 physical agents the dynamics of which can be described by the linearized model of the longitudinal dynamics of an aircraft [45] with

$$A = \begin{bmatrix} -0.277 & 1 & -0.0002 \\ -17.1 & -0.178 & -12.2 \\ 0 & 0 & -6.67 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 6.67 \end{bmatrix}$$

and matching unknown nonlinear functions $f_i(x_i) = ix_{i1} + x_{i2}x_{i3}$. In this example, continuous schemes discussed in Remark 8 are used to solve the problem to reduce chattering phenomena. When approximating the signum functions, we use (49) with $\varepsilon = 1$, and $\varrho = 0.1$.

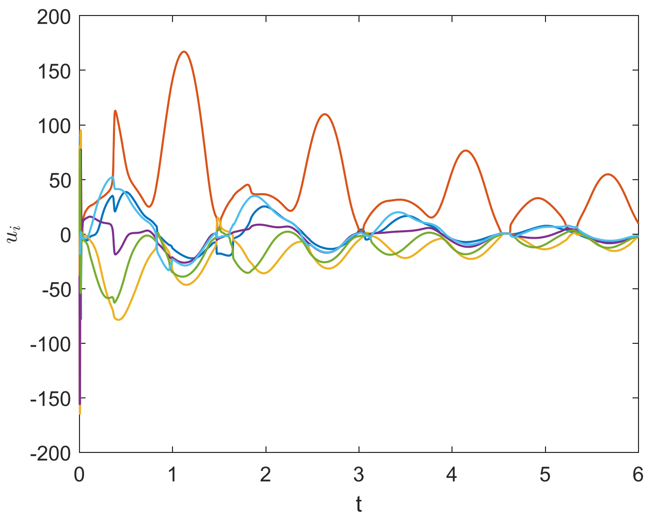

FIGURE 11 Control inputs u_i in continuous counterpart of (10), $i = 1, \ldots, 6$

Firstly, to verify Theorem 1, let $\theta = 2$ and solve LMI (14), one can obtain

$$P = \begin{bmatrix} 1.0117 & -1.4971 & -0.0521 \\ -1.4971 & 21.7860 & 5.6108 \\ -0.0521 & 5.6108 & 2.8395 \end{bmatrix}.$$

Set

$$K = [1.9236, 1.4825, -5.2431]$$

and choose $c = 3$, $\alpha = 3$, $\beta = 0.65$, $\tau_i = 300$, $\pi_i = 100$, $\sigma = 0.6$ following Algorithm 1. Then all conditions of Theorem 1 are satisfied. After implementing continuous counterpart of protocol (10), the states of the agents are visualized in Figure 9, the component-wise values of pseudo converge errors $\hat{W}_i^e$ are provided in

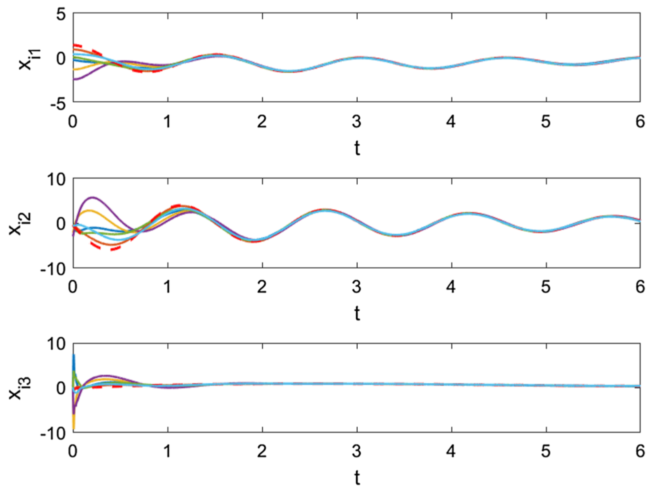

FIGURE 12 States of agents x_i (solid lines) and the average of r_i (dashed line), $i = 1, \ldots, 6$

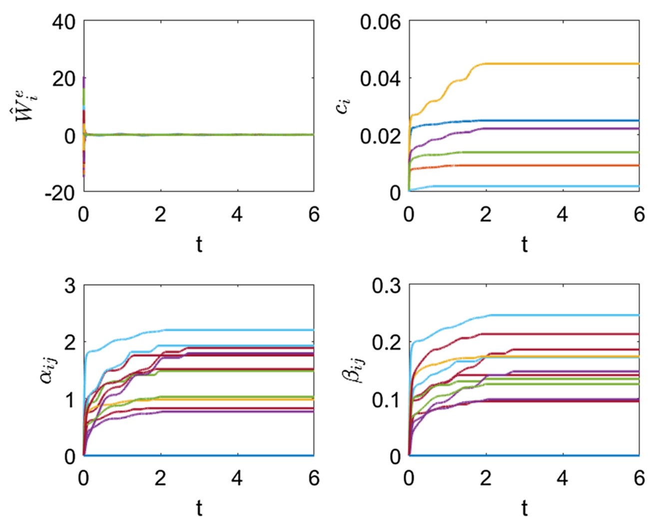

FIGURE 13 Component-wise values of $\hat{W}_i^e$ (upper left), and adaptive coupling weights c_i (upper right), α_{ij} (lower left) and β_{ij} (lower right), $i, j = 1, \ldots, 6$

Figure 10 and the continuous control inputs are shown in Figure 11.

Secondly, to verify Theorem 2, continuous counterpart of dynamic protocol (34) is implemented for the network system with the same initial conditions and NN parameters as above and $\kappa_i = \mu_{ij} = \nu_{ij} = 0.1$ and $T_1 = T_2 = 0.6$. Then the states of all agents are provided in Figure 12. The component-wise values of $\hat{W}_i^e$ and dynamical coupling weights $c_i, \alpha_{ij}, \beta_{ij}$ are shown in Figure 13, and finally Figure 14 shows the continuous control inputs.

It can be seen that both the simulation examples validate the theoretical results.

6 | CONCLUSION

In this paper, a DAT problem for a class of uncertain agents is considered, where the dynamics of the agents contain matching unknown nonlinearities, and the dynamics of the external signals are also unknown. Under some mild assumptions, a robust scheme is designed for the network, where, for each agent, a local filter to be communicated is designed to estimate the average of external signals, and a neural network is embedded to approximate its inherent unknown nonlinear term. State-dependent coupling weights are designed for each communication channel. Adaption parameters are further used to eliminate the dependence of the design procedure to global information like the algebraic connectivity of the communication topology, the NN approximation error, and the external signals. Two numerical examples are given to verify the theoretical results. Future works along the same line will focus on the design and analysis of finite-time protocols and the extensions to networks with directed communication topologies.

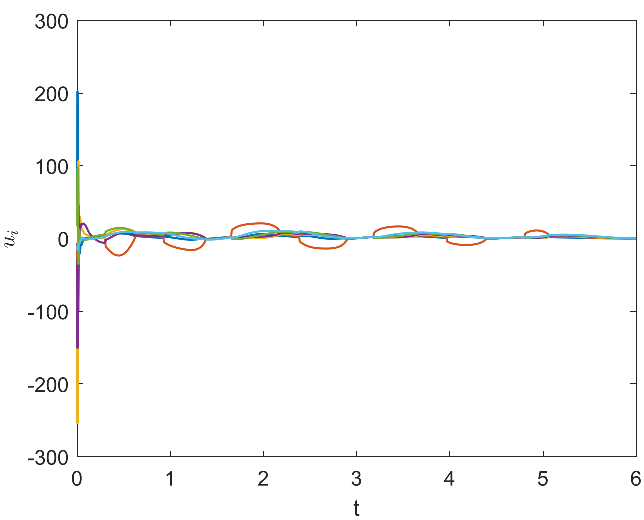

FIGURE 14 Control inputs u_i in continuous counterpart of (34), $i = 1, \ldots, 6$

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REFERENCES

1. Y. Cao et al., *An overview of recent progress in the study of distributed multi-agent coordination*, IEEE Trans. Ind. Inform. **9** (2013), no. 1, 427–438.
2. A. Kantamneni et al., *Survey of multi-agent systems for microgrid control*, Eng. Appl. Artif. Intell. **45** (2015), 192–203.
3. J. Qin et al., *Recent advances in consensus of multi-agent systems: A brief survey*, IEEE Trans. Ind. Electron. **64** (2017), no. 6, 4972–4983.
4. J. Wang, D. Cheng, and X. Hu, *Consensus of multi-agent linear dynamic systems*, Asian J. Control **10** (2008), no. 2, 144–155.
5. Z.-H. Guan et al., *Synchronization of complex dynamical networks with time-varying delays via impulsive distributed control*, IEEE Trans. Circuits Syst. I, Reg. Papers **57** (2010), no. 8, 2182–2195.
6. C. P. Chen, C.-E. Ren, and T. Du, *Fuzzy observed-based adaptive consensus tracking control for second-order multiagent systems with heterogeneous nonlinear dynamics*, IEEE Trans. Fuzzy Syst. **24** (2016), no. 4, 906–915.
7. W. Yu, Y. Li, G. Wen, X. Yu, and J. Cao, *Observer design for tracking consensus in second-order multi-agent systems: Fractional order less than two*, IEEE Trans. Autom. Control **62** (2017), no. 2, 894–900.
8. W. Xu et al., *Event-triggered schemes on leader-following consensus of general linear multiagent systems under different topologies*, IEEE Trans. Cybern. **47** (2017), no. 1, 212–223.
9. H. Haghshenas, M. A. Badamchizadeh, and M. Baradarannia, *Containment control of heterogeneous linear multi-agent systems*, Automatica **54** (2015), 210–216.
10. G. Wen et al., *Containment of higher-order multi-leader multi-agent systems: A dynamic output approach*, IEEE Trans. Autom. Control **61** (2016), no. 4, 1135–1140.
11. F. Chen et al., *Distributed average tracking for reference signals with bounded accelerations*, IEEE Trans. Autom. Control **60** (2014), no. 3, 863–869.
12. F. Chen et al., *Distributed average tracking of networked Euler-Lagrange systems*, IEEE Trans. Autom. Control **60** (2015), no. 2, 547–552.
13. S. Ghapani et al., *Distributed average tracking for double-integrator multi-agent systems with reduced requirement on velocity measurements*, Automatica **81** (2017), 1–7.

14. Y. Zhao et al., *Distributed average tracking for multiple signals generated by linear dynamical systems: An edge-based framework*, Automatica **75** (2017), 158–166.
15. H. Hong et al., *Fixed-time connectivity-preserving distributed average tracking for multiagent systems*, IEEE Trans. Circuits Syst. II Exp. Briefs **64** (2017), no. 10, 1192–1196.
16. Y. Zhao et al., *Finite-time distributed average tracking for second-order nonlinear systems*, IEEE Trans. Neural Netw. Learn. Syst. **30** (2019), no. 6, 1780–1789.
17. S. Ghapani, S. Rahili, and W. Ren, *Distributed average tracking of physical second-order agents with heterogeneous unknown nonlinear dynamics without constraint on input signals*, IEEE Trans. Autom. Control **64** (2019), no. 3, 1178–1184.
18. F. Chen and W. Ren, *A connection between dynamic region-following formation control and distributed average tracking*, IEEE Trans. Cybern. **48** (2018), no. 6, 1760–1772.
19. S. Rahili and W. Ren, *Distributed continuous-time convex optimization with time-varying cost functions*, IEEE Trans. Autom. Control **62** (2017), no. 4, 1590–1605.
20. J. Hu, J. Yu, and J. Cao, *Distributed containment control for nonlinear multi-agent systems with time-delayed protocol*, Asian J. Control **18** (2016), no. 2, 747–756.
21. D. P. Spanos, R. Olfati-Saber, and R. M. Murray, *Dynamic Consensus on Mobile Networks*, IFAC World Congr. Citeseer, Czech Republic: Prague, 2005, pp. 1–6.
22. S. S. Kia, J. Cortés, and S. Martínez, *Distributed event-triggered communication for dynamic average consensus in networked systems*, Automatica **59** (2015), 112–119.
23. S. S. Kia et al., *Tutorial on dynamic average consensus: The problem, its applications, and the algorithms*, IEEE Control Syst. Mag. **39** (2019), no. 3, 40–72.
24. A. Das and F. L. Lewis, *Distributed adaptive control for synchronization of unknown nonlinear networked systems*, Automatica **46** (2010), no. 12, 2014–2021.
25. A. Das and F. L. Lewis, *Cooperative adaptive control for synchronization of second-order systems with unknown nonlinearities*, Int. J. Robust Nonlinear Control **21** (2011), no. 13, 1509–1524.
26. H. Zhang and F. L. Lewis, *Adaptive cooperative tracking control of higher-order nonlinear systems with unknown dynamics*, Automatica **48** (2012), no. 7, 1432–1439.
27. S. El-Ferik, A. Qureshi, and F. L. Lewis, *Neuro-adaptive cooperative tracking control of unknown higher-order affine nonlinear systems*, Automatica **50** (2014), no. 3, 798–808.
28. Z. Peng et al., *Distributed neural network control for adaptive synchronization of uncertain dynamical multiagent systems*, IEEE Trans. Neural Netw. Learn. Syst. **25** (2014), no. 8, 1508–1519.
29. Z. Peng, J. Wang, and D. Wang, *Distributed containment maneuvering of multiple marine vessels via neurodynamics-based output feedback*, IEEE Trans. Ind. Electron. **64** (2017), no. 5, 3831–3839.
30. C. P. Chen et al., *Adaptive consensus control for a class of nonlinear multiagent time-delay systems using neural networks*, IEEE Trans. Neural Netw. Learn. Syst. **25** (2014), no. 6, 1217–1226.
31. D. Yue et al., *Neuro-adaptive consensus strategy for a class of nonlinear time-delay multi-agent systems with an unmeasurable high-dimensional leader*, IET Control Theory Appl. **13** (2018), no. 2, 230–238.
32. G. Wen et al., *Robust neuro-adaptive containment of multileader multiagent systems with uncertain dynamics*, IEEE Trans. Syst. Man Cybern. Syst. **49** (2019), no. 2, 406–417.
33. Z. Li et al., *Distributed tracking control for linear multiagent systems with a leader of bounded unknown input*, IEEE Trans. Autom. Control **58** (2013), no. 2, 518–523.
34. Y. Zhao et al., *Distributed average tracking for lipschitz-type of nonlinear dynamical systems*, IEEE Trans. Cybern. **49** (2019), no. 12, 4140–4152.
35. H. K. Khalil and J. W. Grizzle, *Nonlinear Systems*, Vol. **3**, Prentice Hall, Upper Saddle River, NJ, 2002.
36. M. H. Stone, *The generalized weierstrass approximation theorem*, Math. Mag. **21** (1948), no. 5, 237–254.
37. S. El-Ferik, H. A. Hashim, and F. L. Lewis, *Neuro-adaptive distributed control with prescribed performance for the synchronization of unknown nonlinear networked systems*, IEEE Trans. Syst. Man, Cybern. Syst. **48** (2018), no. 12, 2135–2144.
38. Y. Zhao et al., *Adaptive consensus for multiple nonidentical matching nonlinear systems: An edge-based framework*, IEEE Trans. Circuits Syst. II Exp. Briefs **62** (2015), no. 1, 85–89.
39. B. Cheng and Z. Li, *Fully distributed event-triggered protocols for linear multi-agent networks*, IEEE Trans. Autom. Control **64** (2019), no. 4, 1655–1662.
40. E. D. Sontag, Input to state stability: Basic concepts and results, *Nonlinear and Optimal Control Theory*, Springer, Berlin, Heidelberg, 2008, pp. 163–220.
41. F. Lewis, S. Jagannathan, and A. Yesildirak, *Neural Network Control of Robot Manipulators and Non-Linear Systems*, CRC Press, Florida, 1998.
42. G. Wen et al., *Neuro-adaptive consensus tracking of multiagent systems with a high-dimensional leader*, IEEE Trans. Cybern. **47** (2017), no. 7, 1730–1742.
43. A. F. Filippov, *Differential Equations with Discontinuous Righthand Sides: Control Systems*, Vol. **18**, Springer Science & Business Media, Berlin, 2013.
44. C. Edwards and S. Spurgeon, *Sliding Mode Control: Theory and Applications*, Crc Press, Florida, 1998.
45. B. S. Heck and A. A. Ferri, *Application of output feedback to variable structure systems*, AIAA J. Guid. Control Dyn. **12** (1989), no. 6, 932–935.

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